Operational Limits in Tokamaks

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Introduction
Optimisation of Tokamak Performance

\[ P_{\text{fusion}} \propto \langle p^2 \rangle V \propto \beta^2 B^4 V \]

\[ \beta = \frac{\langle p \rangle}{B^2 / (2\mu_0)} \]

Increase in tokamak performance constraint by operational limits (= instabilities)
Hugill Diagram

[Image of a graph showing data points and lines indicating different wall conditions and power levels.]

[J Rapp et al. 1999 Proc. 26th EPS, Maastricht, ECA 23J 665]
Example: Beta + Density Limit + Disruption

Electron density is continuously ramped up (strong gas fuelling)

Good confinement before \( t = 1.26 \text{ s} \)

Confinement degradation caused by onset of an (neoclassical) tearing mode (NTM, practical beta limit)

\( N^{GW} \sim 2 \) due to good wall conditioning (fresh siliconisation)

Disruption after MARFE onset
Radiation Limits
Radiation Processes and Radiation Instabilities

- **Bremsstrahlung**
  \[ P_{\text{br}} \sim Z^2 n_e n_Z T_e^{1/2} \]
  Balanced by heating power

- **Cyclotron radiation**
  \[ P_c = \frac{e^4}{(3\pi\varepsilon_0 m_e^3 c^3)} B^2 n_e T_e \]
  Re-absorption, plasma is optical thick at fundamental frequency

- **Line radiation**
  \[ P_R = R(T_e) n_e n_l \]
  Instability can develop due to shape of radiation function: radiation → drop of \( T_e \) → enhanced radiation ...

Density Limit - Radiative Collapse

- Poloidally symmetric radiation from the edge
- Line radiation from low-Z impurities
- Edge temperature drops with increasing edge density $\Rightarrow$ radiation increases
- Density limit defined by $P_{\text{rad}} = P_{\text{heat}}$

$$n_e^{\text{crit}} \propto \left( \frac{P_{\text{heat}}}{(Z_{\text{eff}} - 1)} \right)^{1/2}$$

$\Rightarrow$ Density limit can be enhanced by high heating power and reduction of impurities in the plasma
Density Limit - MARFES

MARFE = Multifaceted Asymmetric Radiation From the Edge

MARFE occurs on the high field side (HFS) of the torus

\[ T_e \sim \text{few eV} \]

\[ n_e \sim \text{several } 10^{20}\text{m}^{-3} \]

Onset of MARFE is connected to influx of recycling particles from the wall

Greenwald limit: \( \bar{n}_e \sim \kappa \bar{j} \)

⇒ Increase of the density limit with controlled displacement of the plasma column, i.e. reduced recycling possible
High-Z Impurity Accumulation

Cooling of the plasma centre by radiation

⇒ Drop in temperature, flat profile, loss of neo-classical temperature screening ($\propto T'$)
⇒ Increase of accumulation rate
⇒ Displacement of the plasma current, reversed magnetic shear in the centre
⇒ Double tearing modes, internal disruptions
MHD Stability Limits
$q_a$ Limit

$m=2$ tearing mode when resonant $q=2$ surface is inside the plasma

Kink instability if resonant surface lies outside

$q_a = 5 \ a^2 \ B_t \ / \ (R I_p) > 2$  \quad (cylindrical)

Plasma current has upper limit for a given toroidal magnetic field
Ideal Beta Limit („Troyon Limit“)

Maximum beta depends on pressure profile, current profile, and plasma shape (elongation, triangularity)

$$\beta_t = 2\mu_0 \langle p \rangle / B_t^2$$

Numerical calculations with respect to ballooning modes, Mercier limit, and $n = 1$ kink modes with optimised pressure and current profiles (Troyon 1984)

$n = 1$ kink mode limits the achievable beta:

$$\beta_{t\text{max}} = C I_p / (aB_t)$$

Normalised beta:

$$\beta_{N\text{max}} = \beta_{t\text{max}} / (I_p / aB_t) \approx 2.8$$

For circular shape and large aspect ratio:

$$\beta_{p\text{max}} = 0.14 (R/a) q_a$$

Beta limit *transiently* reached, in *stationary* discharges below ideal prediction
Tearing Modes

Tearing instability is driven by radial current profile gradients.

Growth of tearing modes determined by *tearing parameter*, mode is unstable if $\Delta' > 0$

\[
\Delta'(w) = \frac{1}{B_r} \left( \frac{\partial B_r}{\partial r} \right) |_{r_s-w/2}^{r_s+w/2}
\]

Size of magnetic island

\[
\frac{dw}{dt} \cong \frac{\eta}{2\mu_0} \Delta'(w)
\]

Growth time

\[
\gamma^{-1} \approx \tau_R^{3/5} \tau_A^{2/5}
\]
Double Tearing Modes (DTM)

- Non-monotonic $q$ profiles
- Off-axis current drive
- Impurity accumulation during plasma start-up
- Reversed magnetic shear attractive for tokamak Operation
- Transport barriers
- Non-inductive current

DTMs: large spatial extension; rapid growth; mode coupling / overlap; minor and major disruptions; off-axis sawteeth
• Confinement drop is correlated to the onset of a 3/2 **neo-classical tearing mode**
• $\Delta' < 0$, a so-called **seed island** is required, sawtooth crash triggers mode
• $\beta_N$ scales approx. linear with the poloidal ion gyro radius $\Rightarrow$ low threshold in ITER
Locked Modes

- Stationary magnetohydrodynamic perturbation
- Plasma rotation

\[ \omega_e^* = \frac{m}{e n_e B r} p' \]

Radial electric field \( v_\phi = \frac{E_r}{B_\theta} \)

Toroidal rotation due to momentum transfer (NBI)

- Slowing down of mode rotation due to friction forces
- Mode locks to wall (rotation stops)
- Locked mode enhances transport / leads to disruption
- Detection with specially designed magnetic diagnostic or from profile \((T_e, n_e)\) measurements
- Momentum transfer and sheared rotation counteract mode locking
Error Fields

- Non-axisymmetric error fields originate from non-ideal coil alignments and current feeders
- Locked modes \((m/n=2/1)\) are excited above a critical field threshold
- Threshold depends on density, toroidal field, configuration, plasma rotation, beta, ...
- Critical error fields are in the order \(B_r/B_t \approx 10^{-5} \ldots 10^{-4}\)
- External error field correction coils are needed to cancel the intrinsic error field

\[ B_{\text{crit}} \propto n_e \]

[R Buttery et al. 2000 *Nuclear Fusion* 40 807]
Resistive Wall Modes (RWM)

- Beta limited by global external kink mode
- Passively stabilized by ideal conducting close fitting wall
- Resistive wall reduces growth rate
- Active feedback (using saddle coils) and plasma rotation enhance beta limit

[M Okabayashi et al. 2002 PPCF 44 B339]
Vertical Instability of Elongated Plasmas

- Axisymmetric instability with toroidal mode number \( n = 0 \)
- Vertically elongated plasmas are unstable with respect to vertical displacements (VDEs)
- Vertical stability of a circular, large-aspect ratio plasma is determined by the field index (stable if \( n > 0 \))
  \[ n = -\frac{R}{B_v} \frac{dB_v}{dR} \]
- Vertical instability has a large growth rate (inertial time scale)
- Stabilisation by a conducting shell around the plasma (reduces growth rate to resistive time scale) or by active feedback using (external) coils (growth rates up to \(~4000 \text{ s}^{-1}\) can be controlled, TCV 2000)
- Maximum elongation is limited by vertical instability
Disruptions
Runaway Electrons

- Ohmic plasma current requires toroidal electric field
- Electrons are accelerated
- Drift velocity is balanced by collisional force $\rightarrow$ resistivity
  \[ Ee = m_e v_d / \tau_c \]
- Runaway occurs for electrons with $v_e >> v_{th}$ (collisional force $\propto 1/v_e^2$), and at low densities ($\tau_c \propto 1/n$)
- Non-maxwellian distribution function develops
- Large amounts of runaway electrons are created during disruptions
- High-energetic electrons may cause damage / evaporation of first wall
Disruption

Disruption Causes

Two phases:

Energy quench

Current quench

Fast time scale

$L/R \sim ms$

Plasma energy is released to wall

Strong forces due to eddy and halo currents

Disruptivity vs Operational Limits

\[ \text{P de Vries 2009 NF 49 055011} \]
Avoidance / Mitigation of Disruptions

• Goal: reduce forces on the vessel and prevent damage to first wall
• Avoidance of disruptions by
  ▪ Real-time detection of disruption precursor and appropriate action
  ▪ Detection of locked mode and controlled shutdown of the discharge
  ▪ Operation in safe regime far from operational limits
• Mitigation of disruptions by
  ▪ Detection of oncoming disruption with e.g. neural networks etc, and initialisation of soft stop (reduce plasma shaping, heating power, etc)
  ▪ Heating to soften the current quench
  ▪ Massive gas injection (He ... Ar) or pellet/dust injection to mitigate heat loads and forces, and to prevent runaway generation
• Disruptions = Loss of plasma control
Summary

- Tokamak operation and performance is constrained by operational limits.
- Hard (disruption) and soft (confinement deterioration) limits exist:
  - Radiative collapse / MARFEs
  - Impurity accumulation
  - External kink modes ($q_a$)
  - (Neo-classical) tearing modes, double tearing modes
  - Locked modes / error fields
  - Resistive wall modes
  - Vertical instability (VDEs)
- Stationary performance is limited by neo-classical tearing modes.
- Mode stabilisation, error field correction, and prevention/amelioration of disruptions required to optimise performance in fusion experiments.

$\Rightarrow$ A tokamak reactor needs active control of MHD stability.