# MHD instabilities in tokamaks with 3D effects

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# OUTLINE

- **Introduction**
  - 3D effects in tokamaks
  - 3D equilibrium and stability codes
- **Stellarator versus 3D tokamak**
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- **3D equilibrium calculations**
- **Stability studies**
  - tokamak equilibrium with helical core
  - 3D effects of RMP-coils in ASDEX Upgrade
  - 3D effects of Test Blanket Modules (TBMs) in ITER
- **Confinement considerations**

# 3D effects in tokamaks

- **Toroidal field coils: mostly high-n perturbations, e.g. n=16 in AUG**
- **Test blanket moduls: low-n perturbation (n=1) in ITER**
- **RMP coils: low-n perturbations, e.g. n=1, 2 or 4 in AUG**
- **resistive wall: e.g. medium-n perturbation (n=9) in ITER**
- **equilibrium with helical core: low-n perturbation, observed in MAST, TCV, RFX, ….**
- **error fields: small undesignedly or unavoidable non-axisymmetric magnetic fields $(\Delta B/B) \sim 10^{-4}$**

**n= leading toroidal harmonic of the magn. field perturbation**

# 3D equilibrium and stability codes

IPP

## 3D equilibrium codes

| | |
|---|---|
| **NEMEC** | **upgraded version of NEMEC=NESTOR[1]+VMEC [2]code, [1]P. Merkel, [2]S. Hirshman, 3D free-bound. equilib. (assump. of nested flux surf.)** |
| **ANIMEC** | **W. A. Cooper, variant of the VMEC code designed to obtain 3D anisotropic pressure equilibria** |
| **PIES** | **A. Reiman, D. Monticello, 3D equilibrium code, handles islands and stochastic regions** |
| **HINT** | **T. Hayashi, 3D equilibrium code, handles islands and stochastic regions** |

## Coordinate transformation into Boozer coordinates

| | |
|---|---|
| **COTRANS** | **E. Strumberger, coordinate transformation and code interface, contains parts of the JMC code of J. Nuehrenberg and R. Zille** |

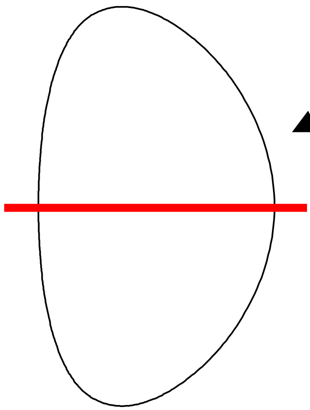
## 3D linear ideal stability codes

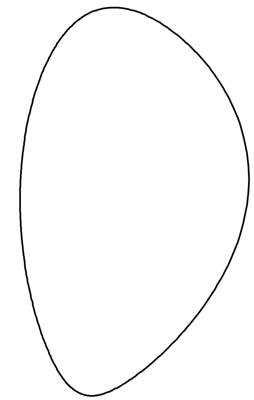
| | |
|---|---|
| **CAS3DN** | **modified version (Non-equidistant radial grid) of the CAS3D code of C. Nuehrenberg** |
| **TERPSICHORE** | **D.V. Anderson, W.A. Cooper, uses finite elements in radial direction and Fourier decomposition in angular variables similar to CAS3D** |

# SYMMETRIES

**axisymmetry**

**B**=**B**(R,Z)





**stellarator symmetry**

**B(r´)=B(r)**

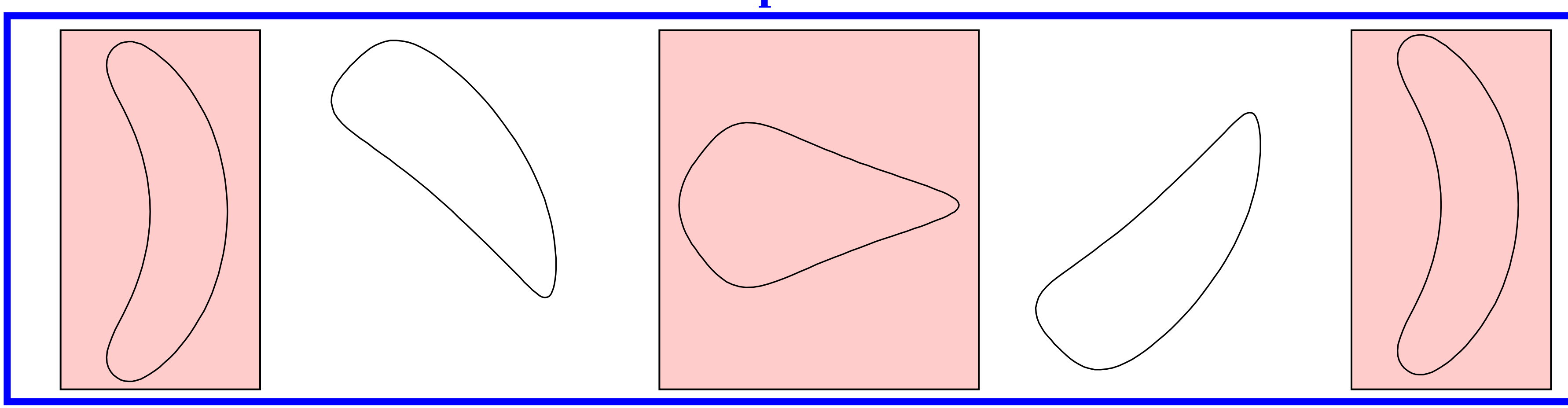
$B_{\mathbf{R}}(\mathbf{r}')= -B_{\mathbf{R}}(\mathbf{r})$, $B_{\boldsymbol{\varphi}}(\mathbf{r}')=B_{\boldsymbol{\varphi}}(\mathbf{r})$, $B_{\mathbf{Z}}(\mathbf{r}')=B_{\mathbf{Z}}(\mathbf{r})$

with $\mathbf{r}=(R,\varphi,Z)$, $\mathbf{r}'=(R,2\pi/N_{\mathbf{p}}-\varphi,-Z)$

**up-down symmetric**

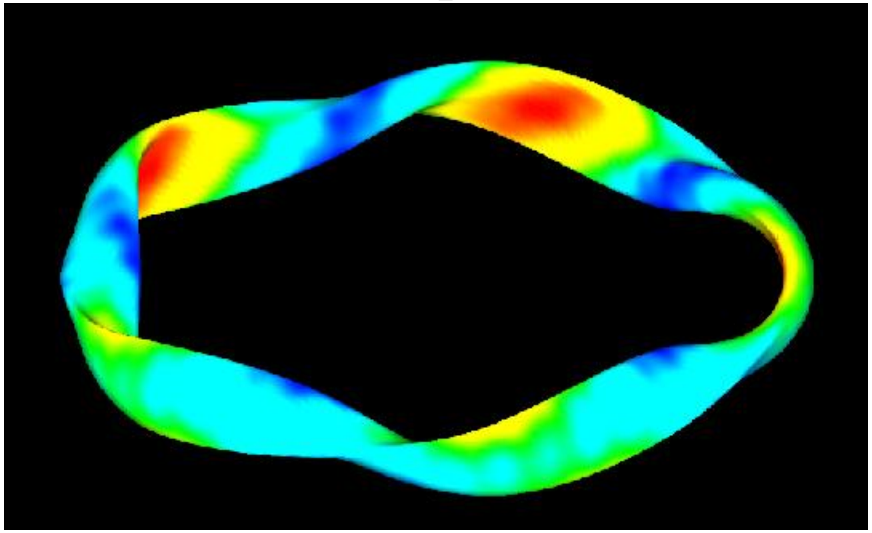
**not stellarator symmetric**

**one period**



# Field periods and mode families

W7-X, 5 field periods ($N_p$=5)



- Depending on the perturbation 3D tokamaks may have $N_p$=1,2,3,…
- Because of the periodicity only those toroidal harmonics contribute to a perturbation which differ by multiples of $N_p$. Such a group is called mode family.

- Depending on $N_p$, there are $1+[N_p/2]$ mode families.

axisym. configuration: $N_p = \infty$, all n's decouple

3D configuration: $N_p = 1$, all n's couple

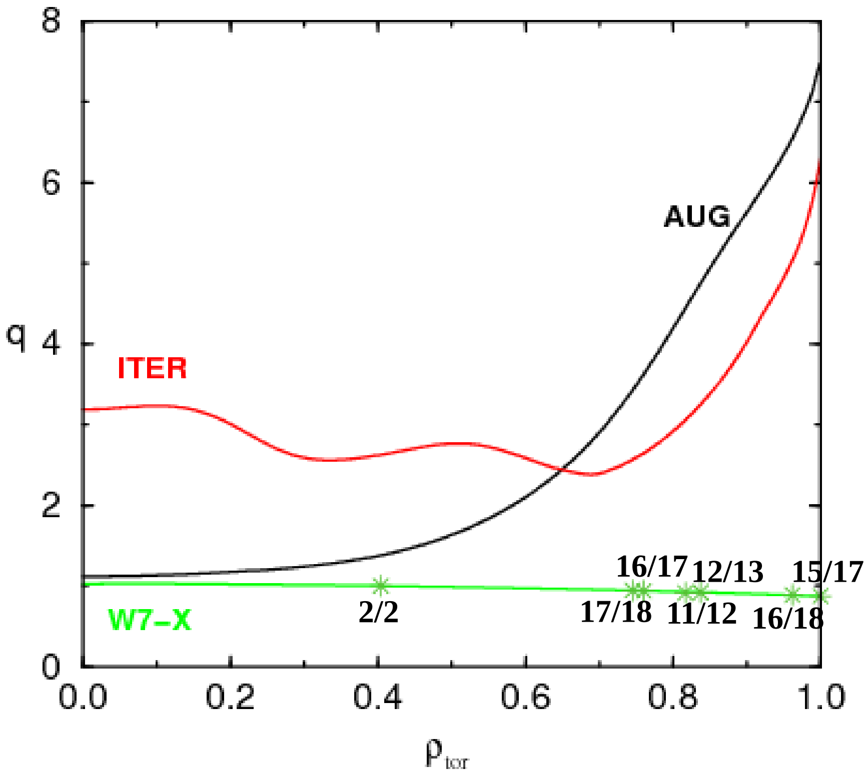
$N_p = 2$, two mode families, n=0,2,4,… and n=1,3,5,…

$N_p = 5$, three mode families, n=0,5,10,15,…

n=1,4,6,9,11,...

n=2,3,7,8,12,13,…

# Safety factor profiles



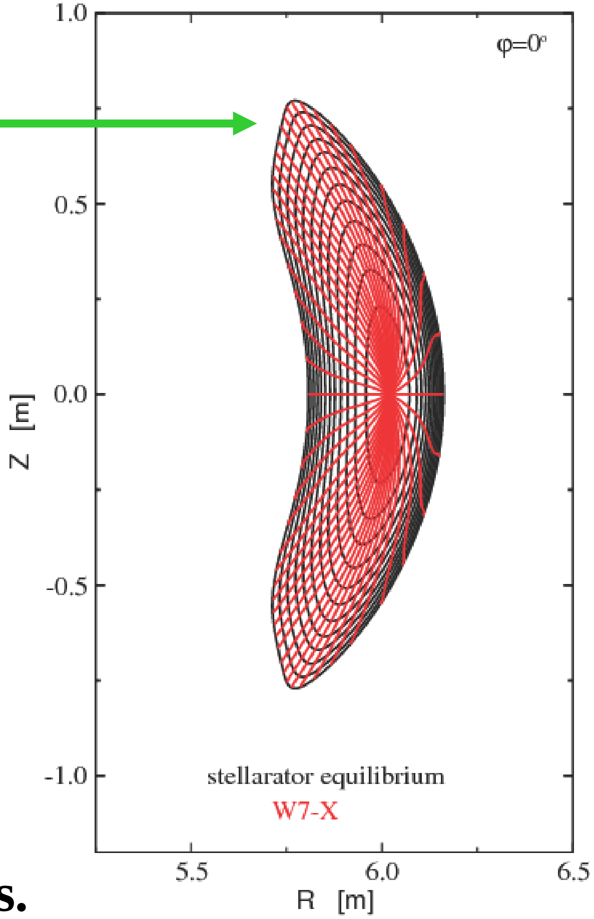

**The safety factor profile of a tokamak contains much more rational surfaces than that of a stellarator.**

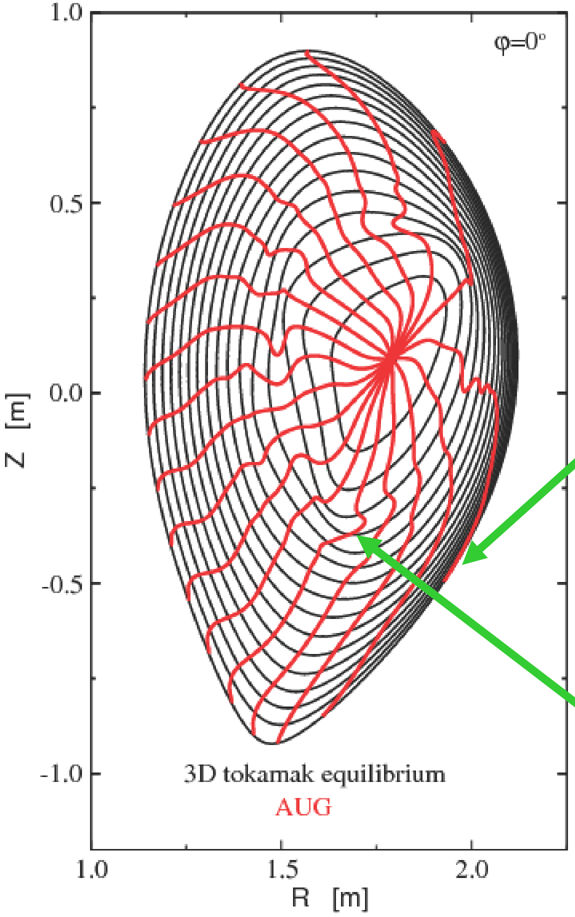
**The stability computations for 3D tokamak configurations require a much higher numerical effort with respect to:**

- **resolution of the radial grid**
- **number of poloidal and toroidal Fourier harmonics which describe the displacement vector**

# Magnetic coordinates (Boozer coordinates)

- The poloidal coordinate lines in a stellarator are smooth lines with small curvature.




- In a tokamak the poloidal coordinate lines become almost parallel to the radial ones for high q-values.
- Their corrugation increases with increasing 3D deformation of the flux surfaces.

# Stellarator versus 3D tokamak: results

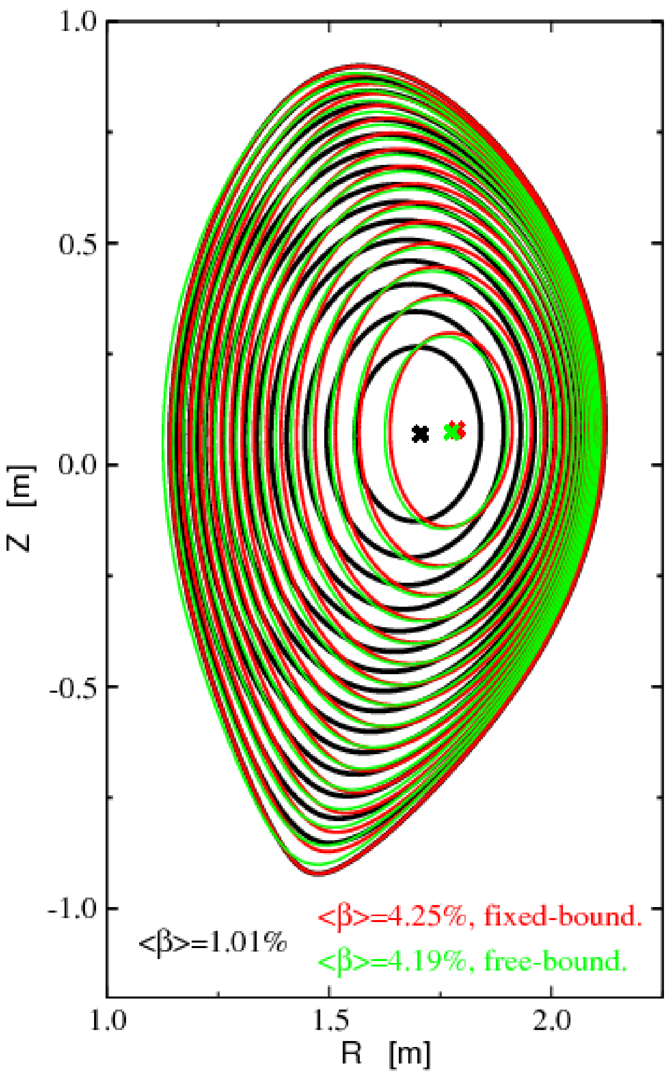
**The numerical effort for**

- **transformation into magnetic coordinates, and**
- **stability studies**

**is much higher for 3D tokamak equilibria than for stellarator equilibria, because of**

- **the numerous rational surfaces, and**
- **the strong bending of the poloidal coordinate lines.**

# Axisymmetric equilibrium calculations

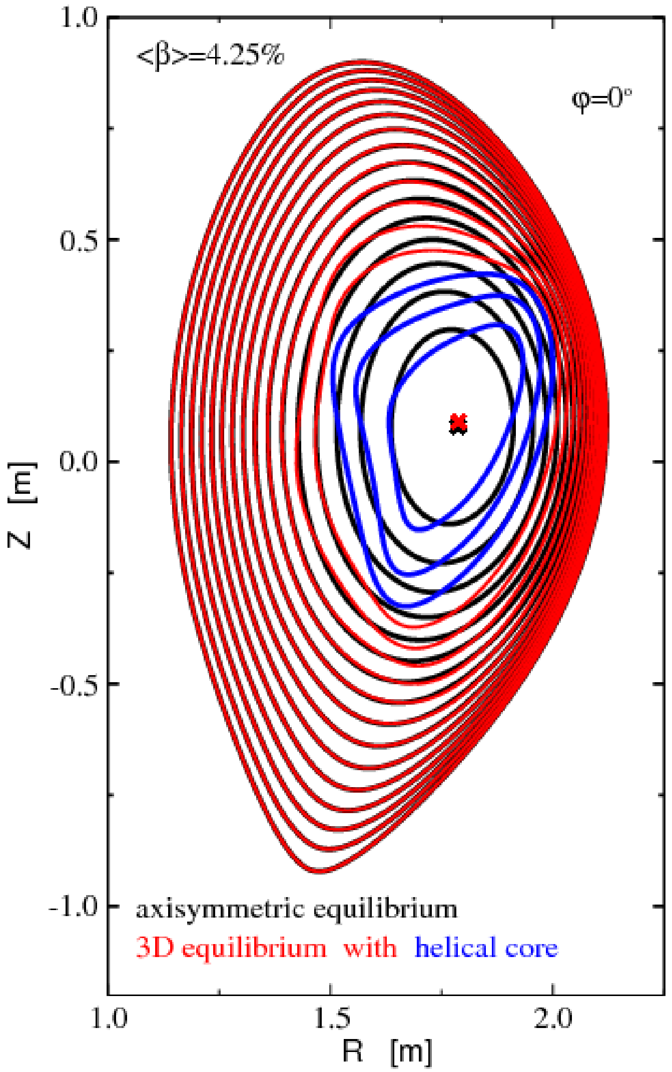



**The NEMEC [1] (NEstor+VMEC) code computes fixed- and free-boundary, low- and high-beta, axisymmetric equilibria with an execellent numerical accuracy (ftol < $10^{-11}$).**
**The stability properties of the equilibria play no role, because the computations are restricted to axisymmetry.**

**Scaling the pressure profile, the plasma beta has been increased. Simultaneously, plasma shape and boundary have been kept fixed roughly by a suitable choice of the coil currents (free-boundary cases).**


**[1] S. Hirshman, W.I. van Rij, and P. Merkel, Comp. Phys. Commun. 43 (1986) 143.**

# 3D equilibrium calculation with axisym. fixed-boundary and initially perturbed magn. axis (2 field periods)




**A helical core with a 3/2 geometry develops.**

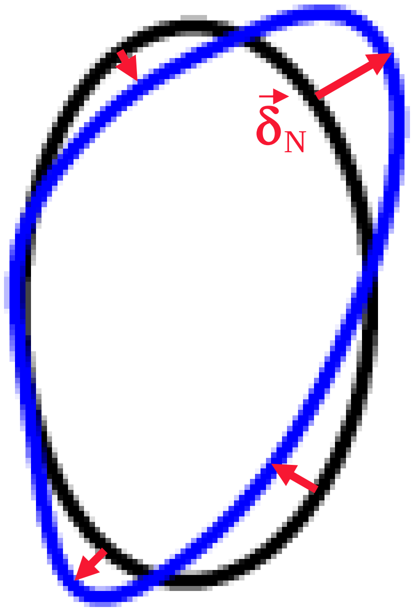
**Comparable results have been found previously, e.g.:**

- **W.A. Cooper et al., PRL 105 (2010) 035003**
  **"The helical equilibrium states resemble saturated internal kink mode structures."**

- **D. Terranova et al., Contrib. Plasma Phys. 50 (2010) 775.**

# Analysis of the 3D fixed-boundary equilibrium solution

measure of the corrugation: $\vec{\delta}_N = \delta_N \vec{n}$




$\vec{\delta}_N$ **: vector pointing in normal direction from the average axisymmetric flux surface to its corresponding 3D surface**

$\delta_N$ **: corresponds to the normal component of the displacement vector in linear MHD stability computations**

$|\delta_N|_{max}(s_i)$ **: max. absolute value of flux surface i**

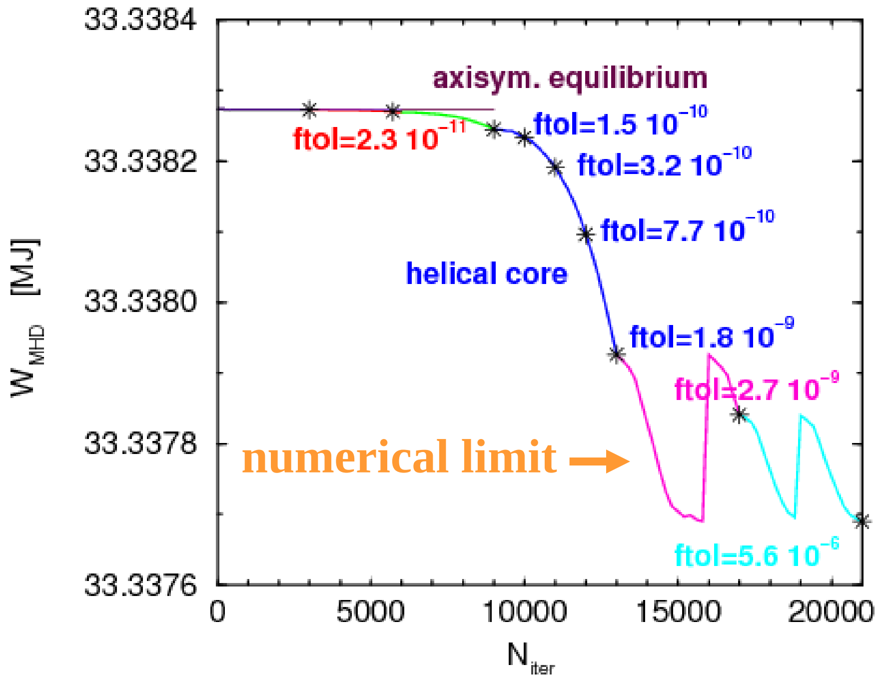
$|\delta_N|^e_{max}$ **: max. absolute value of the equilibrium**

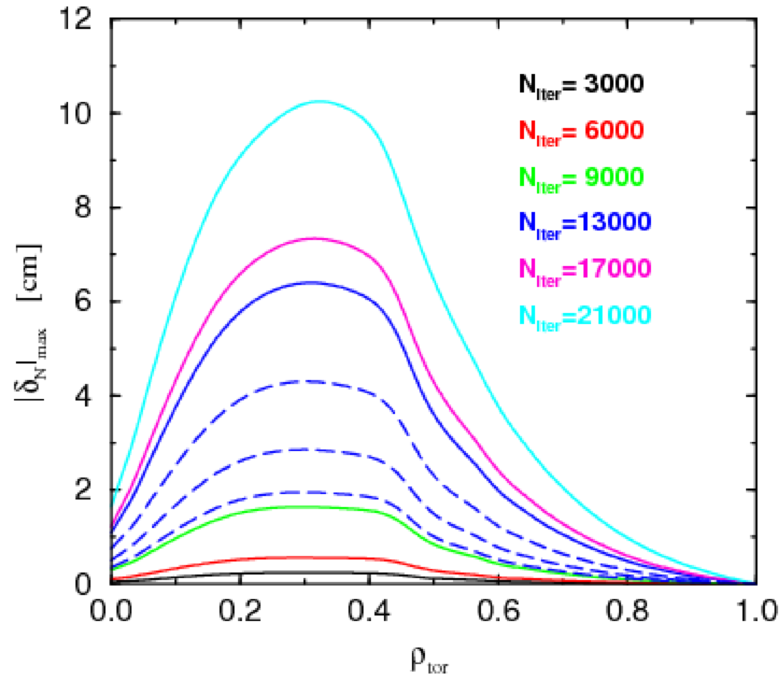
**Fourier representation of $\delta_N$ in magnetic coordinates (Boozer coor.):**

$$\delta_N(s_i) = \sum_{m=0,n=-n_b}^{m_b,n_b} \bar{\delta}^c_{m,n}(s_i)\cos(m\theta + n\phi) + \bar{\delta}^s_{m,n}(s_i)\sin(m\theta + n\phi)$$

# Analysis of the 3D fixed-bound. equilib. solution (cont'd)

**Equilibrium:** $\delta W_{MHD} = 0 \quad \Longleftrightarrow \quad \vec{F} = -\vec{j}\times\vec{B} + \nabla p = 0$



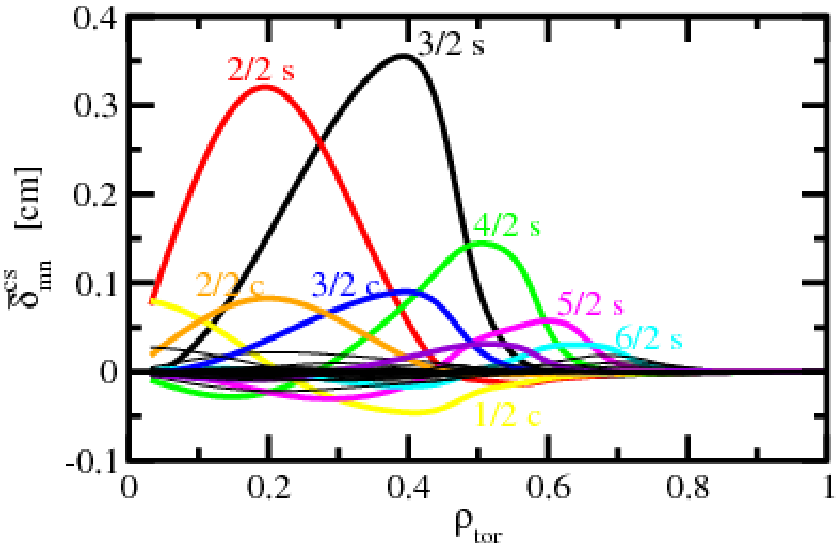

**Energy difference between axisymmetric and 3D equillibrium:**

**$\Delta W_{MHD}$ ~ 300 J**

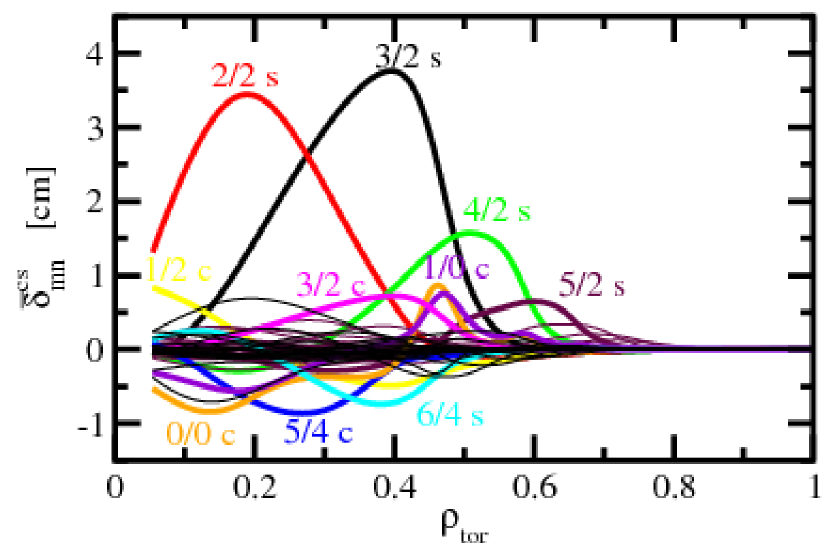
# Analysis of the 3D fixed-bound. equilib. solution (cont'd)

## Fourier representation of $\delta_N$ in magnetic coordinates

**small 3D deformation**




**helical core**




- The Fourier spectrum of $\delta_N$ shows the structure of an internal kink mode.
- With increasing 3D deformation more and more toroidal harmonics, n, are involved.
- Since the equilibrium computation has been performed for a two periodic configuration, only toroidal harmonics n=0,2,4,… appear.

# 3D equilibrium calculations: results

- Equilibria with a helical core and an axisymmetric boundary can be found with 3D NEMEC fixed-boundary computations, if the corresponding axi-symmetric equilibrium is unstable with respect to internal kink modes.

- The corrugation of the flux surfaces reflects the ideal kink structure.

- The energy difference between the axisymmetric and the 3D equilibrium state is very small. Here:

$$\Delta W_{MHD} / W_{MHD} \lesssim 10^{-5}$$

- 3D equilibrium solutions are very sensitive to numerical parameters and accuracy. Those affect the corrugation and, therefore, also the succeeding stability and confinement studies.

→ 3D equilibrium calculations are very demanding.

# 3D equilibrium calculations: results (cont'd)

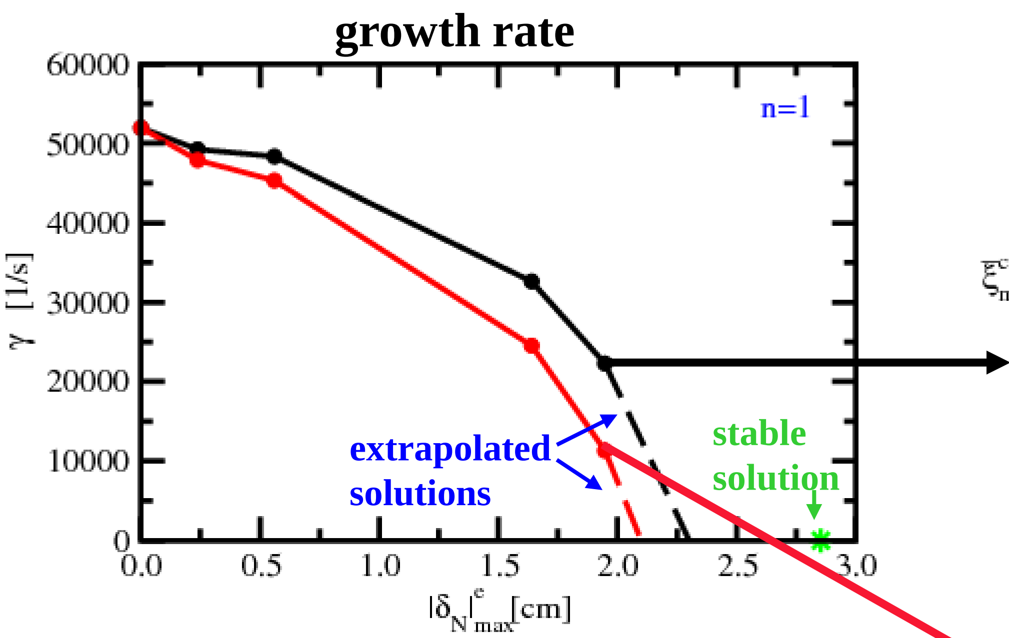
- The chosen number of periods for the equilibrium calculation selects the type of the helical core, e.g. m=1, n=7 for $N_p$=7 in RFX-mod,* or m=3, n=2 for $N_p$= 2 in the considered test case.

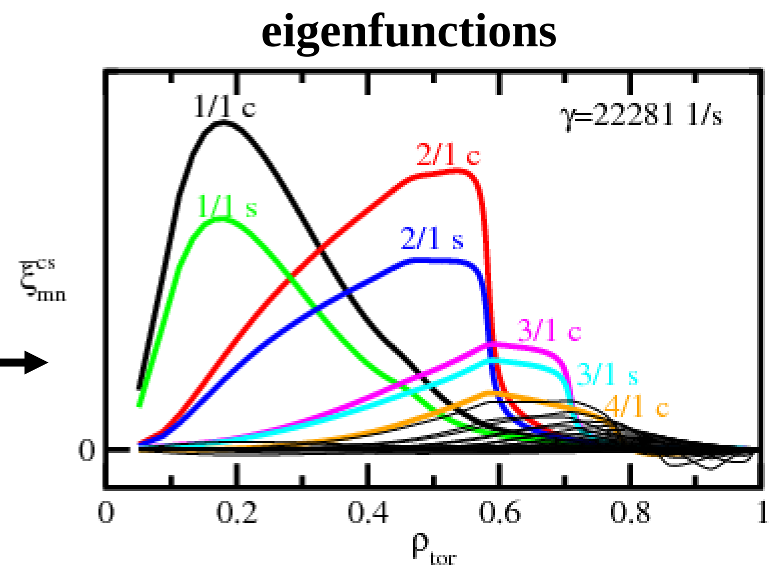
- Assuming a fixed-boundary also restricts the 3D equilibrium solution.

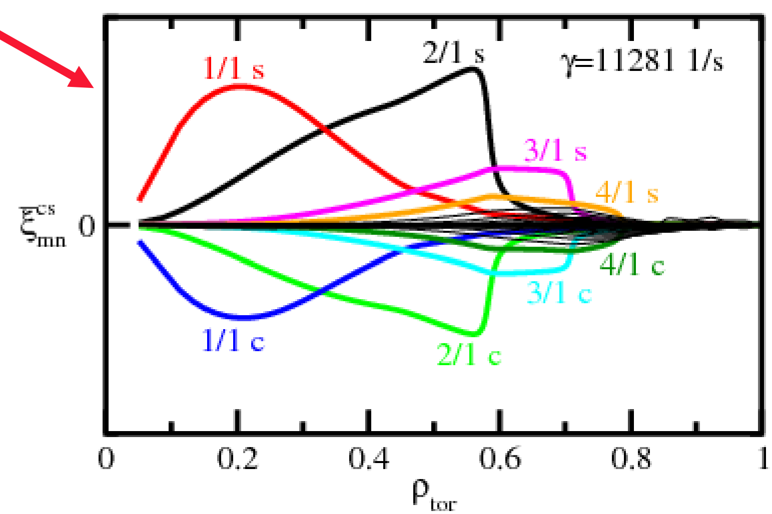
- Poorly, or non-converging solutions indicate that in such cases no 3D equilibrium exists.

*D. Terranova et al., Plasma Phys. Control. Fusion 52 (2010) 124023.

# Stability studies: equilibrium with helical core (fixed-boundary)

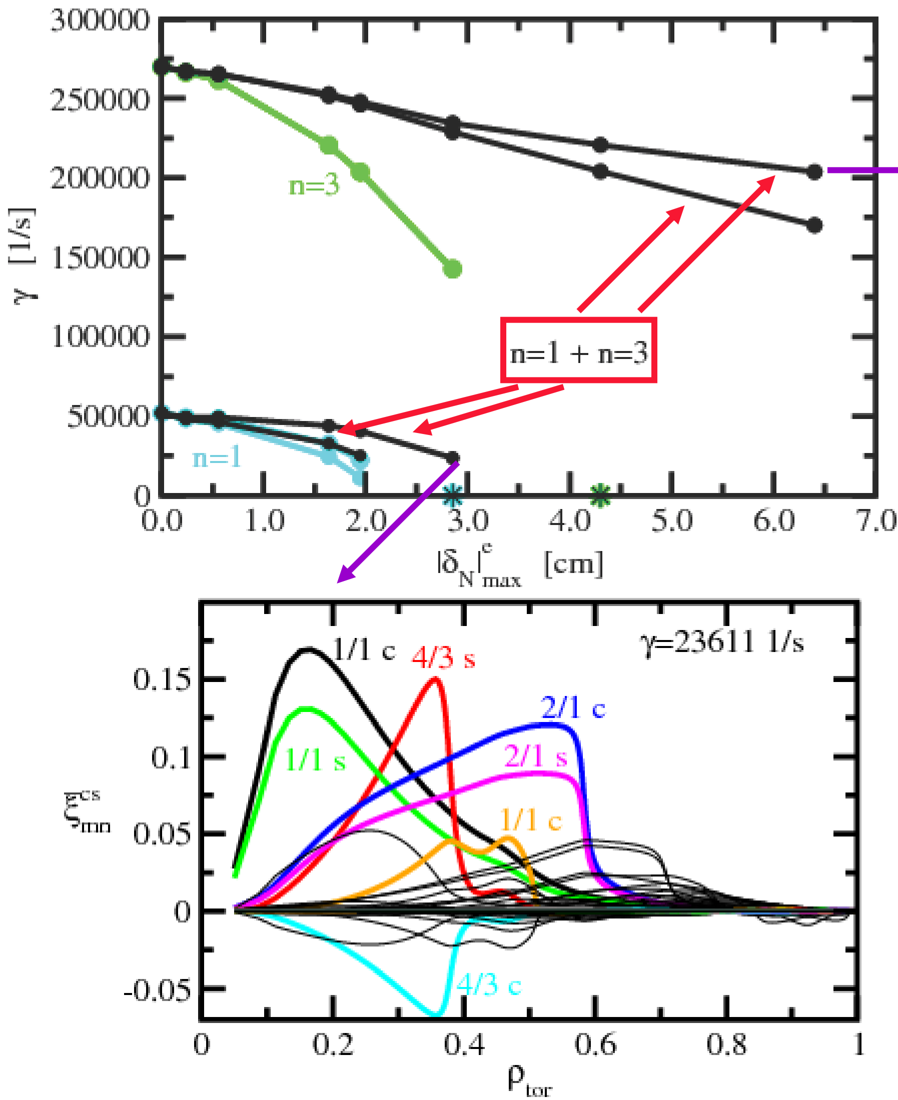


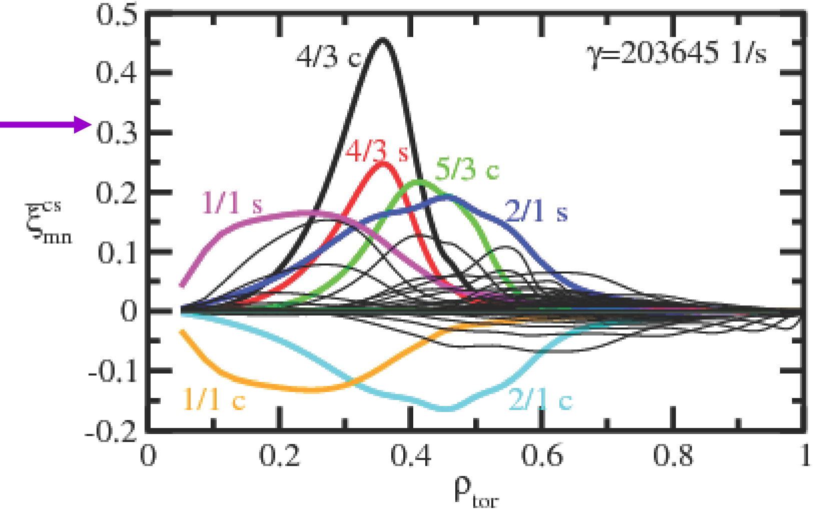




**The 3D deformation of the plasma core has a stabilizing effect on the considered pure n=1 internal kink mode.**
**For a large enough core the mode becomes eventually stabilized.**

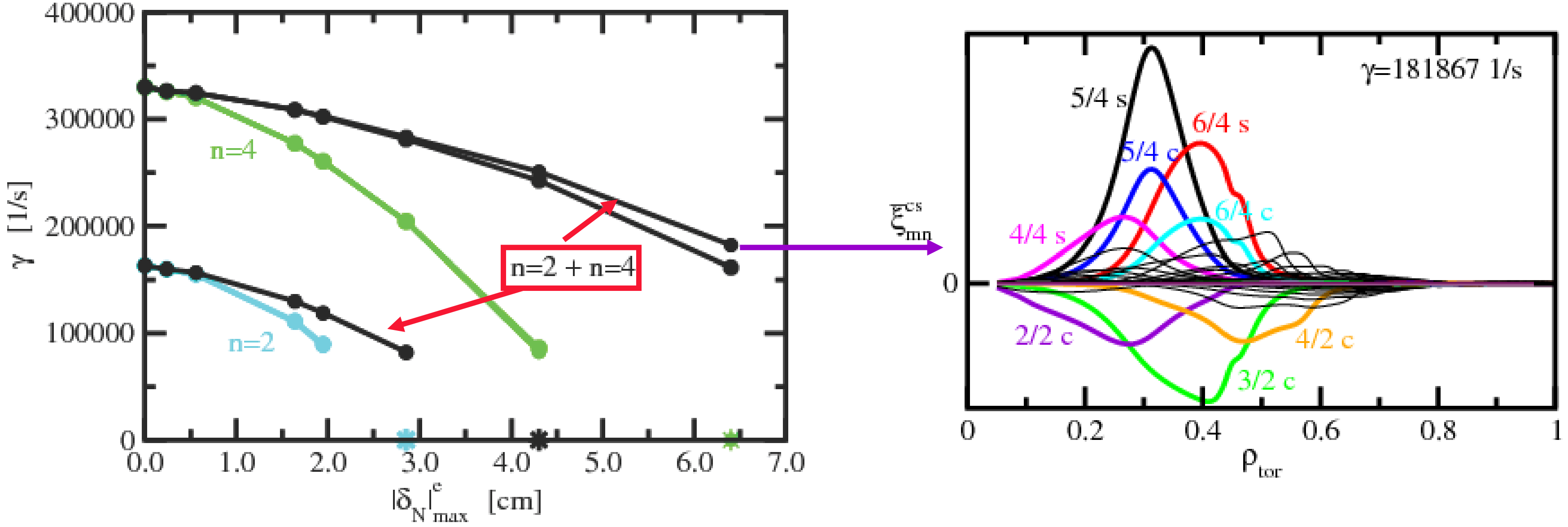
# Stability studies: equilibrium with helical core (cont'd) (fixed-boundary)




**With increasing 3D deformation of the flux surfaces the coupling of the toroidal harmonics become more and more important.**

# Stability studies: equilibrium with helical core (cont'd) (fixed-boundary)




**Here the 3/2 helical core has a greater stabilizing effect on the even than on the odd mode family.**

# Tokamak equilibrium with helical core: results

- **The resulting 3D fixed-boundary equilibrium (3/2 helical core structure) of the considered test case is still internal unstable with respect to**
  - **n=3 with a considerable part of n=1 harmonics,**
  - **n=4 with a considerable part of n=2 harmonics, and**
  - **most likely higher toroidal harmonics.**

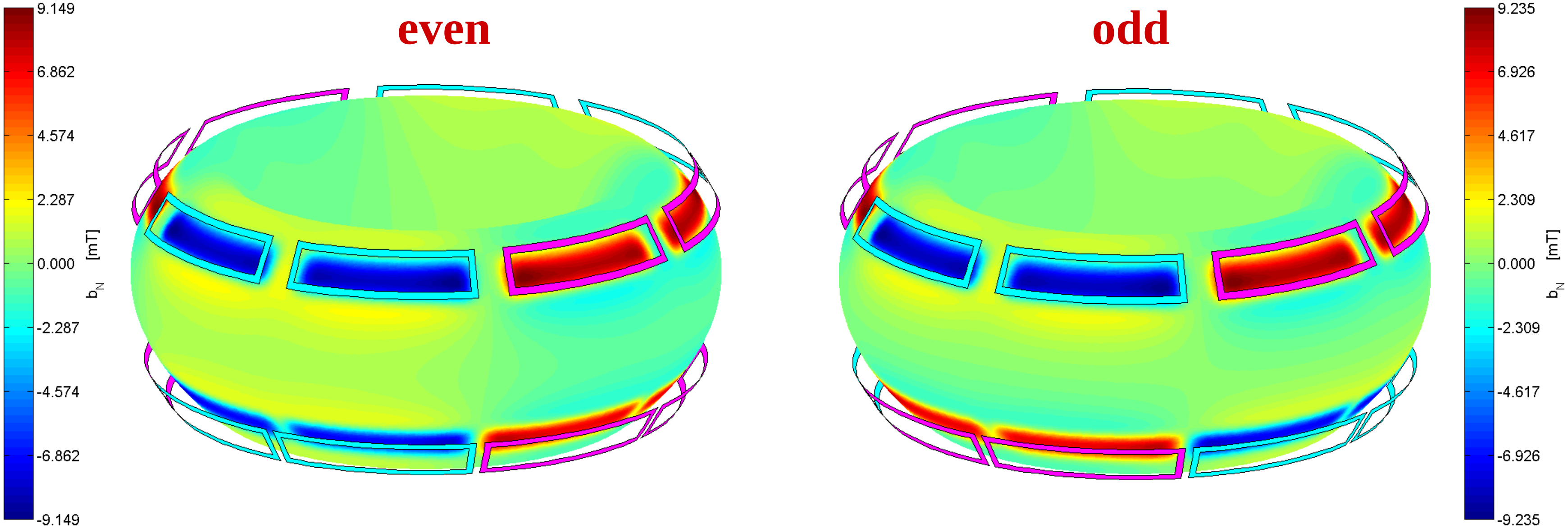
  **This is most likely the main reason for the moderate convergence of the equilibrium solution and its final divergence.**

- **The quality of a 3D equilibrium solution and its stability properties are strongly correlated.**

**3D equilibrium solution ⟷ stability properties**

# RMP coils in AUG

## n=2 perturbation field



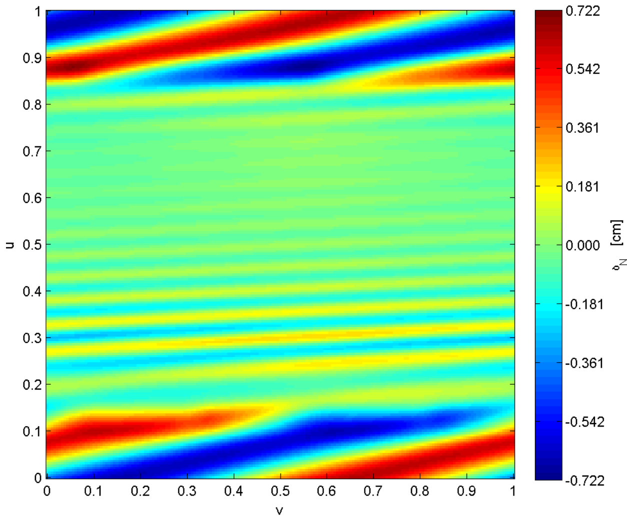

- $(b_N/B_0)_{max} = \pm\, 0.004$

  $b_N$ = normal component of the perturbation field

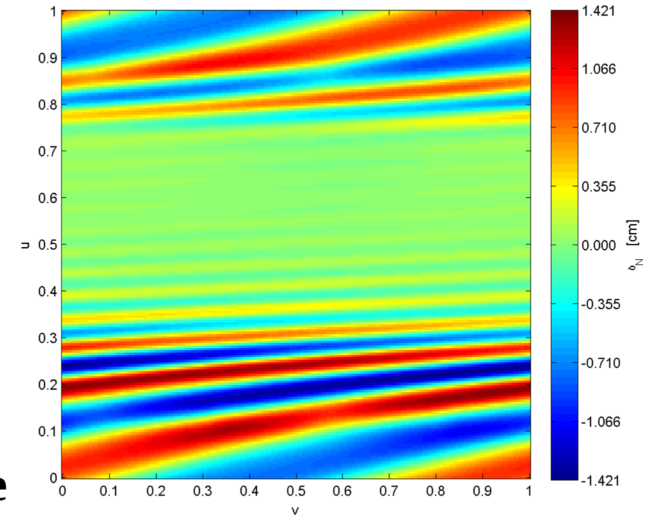
- AUG-type equilibrium, $\langle\beta\rangle = 2$ %

# Corrugation of the flux surfaces

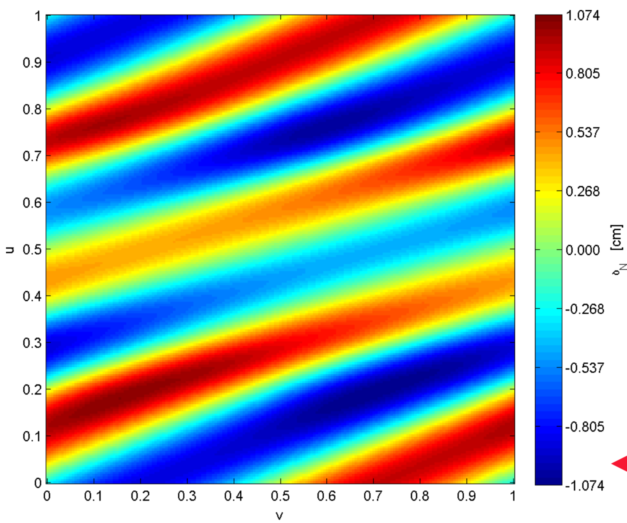
**even** | **odd**



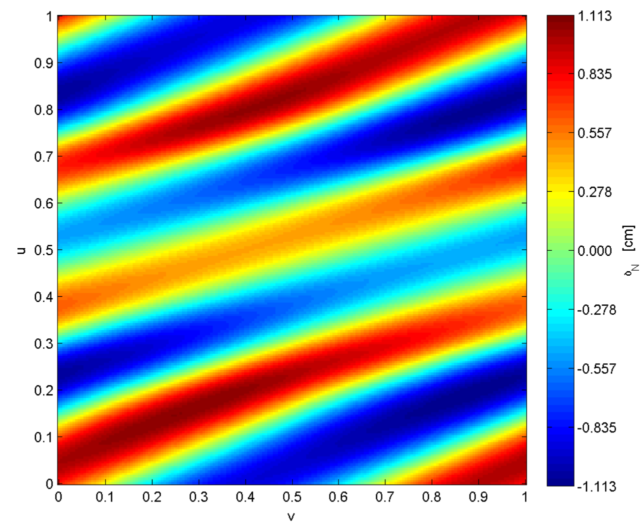

← $q \sim 15/2$, $\rho_{tor}=1$ →




- The corrugation of the flux surfaces is larger for the odd case in the boundary region, but almost the same in the core region.
- There is a shift shift between odd and even case.



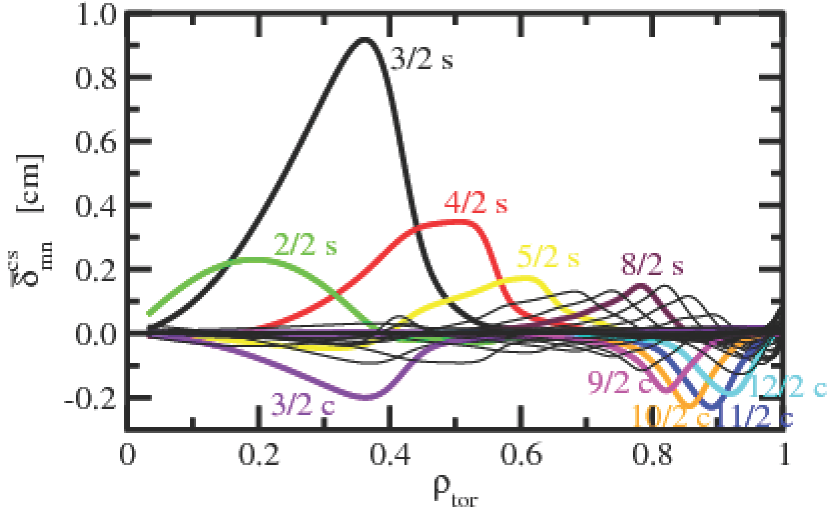

← $q=3/2$, $\rho_{tor} = 0.4$ →



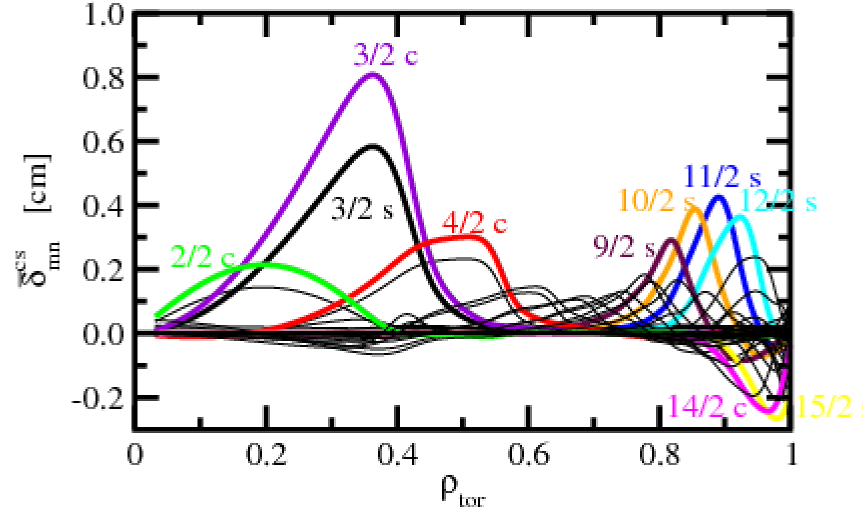
# Corrugation of the flux surfaces (cont'd)

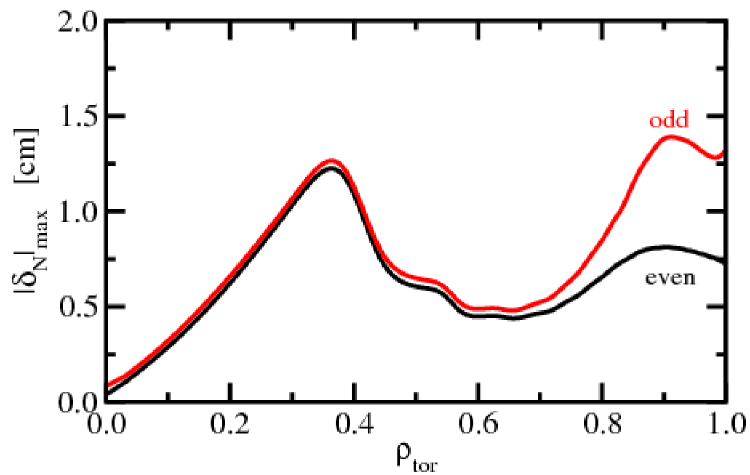
## Fourier spectra (mag. coordinates)

**even**




**odd**

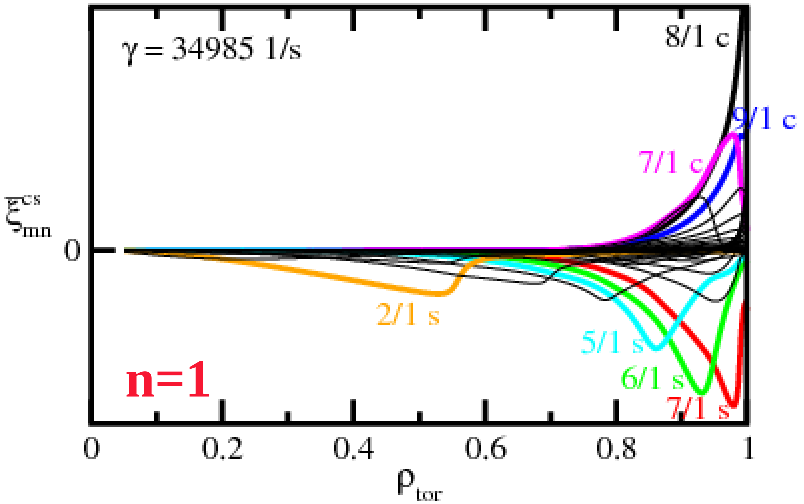


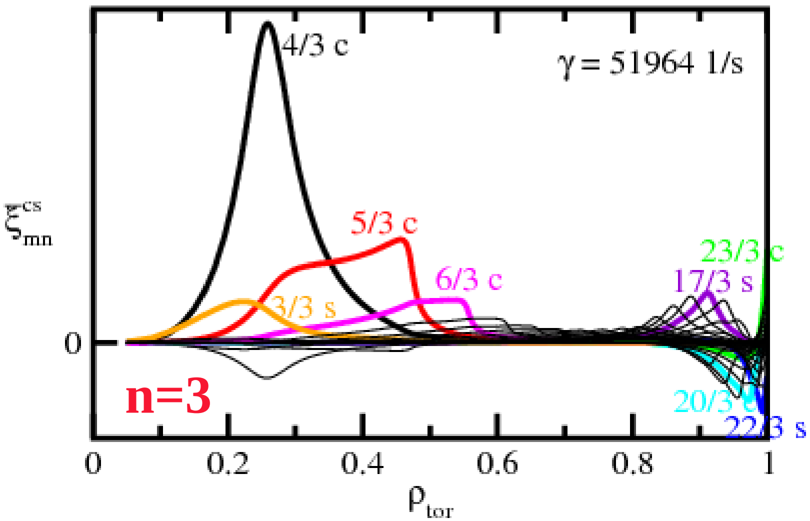




**maximum corrugation as function of $\rho_{tor}$**

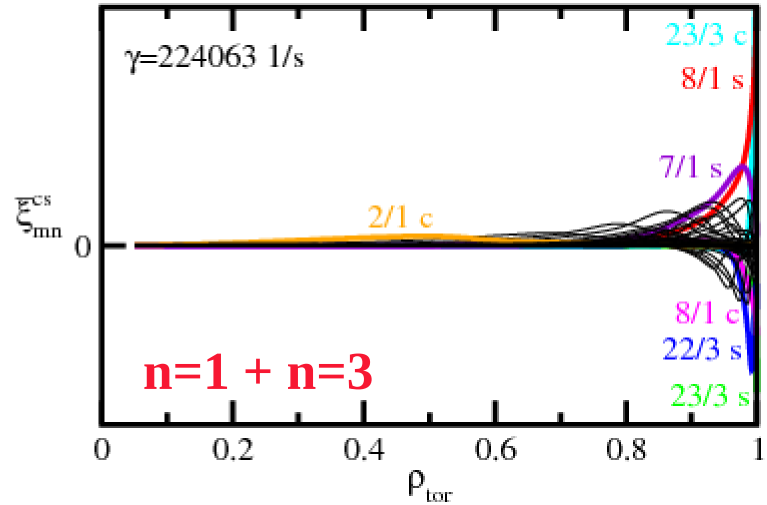
# Stability properties (n=1, 3)

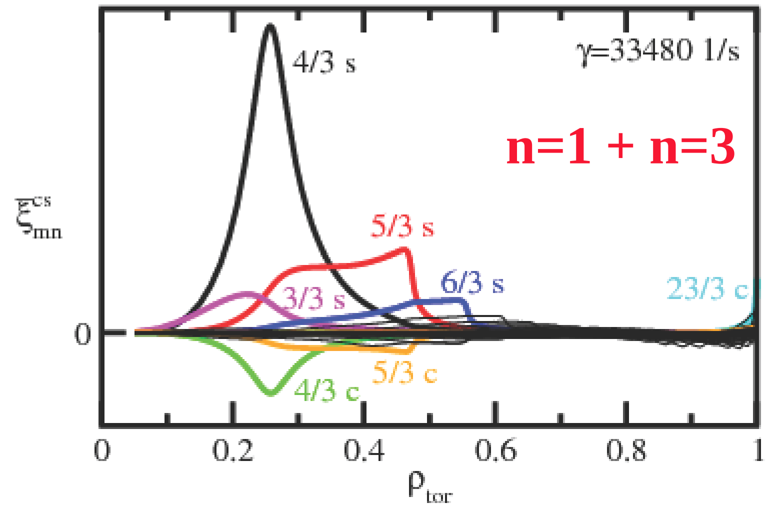
## axisym. equilibrium



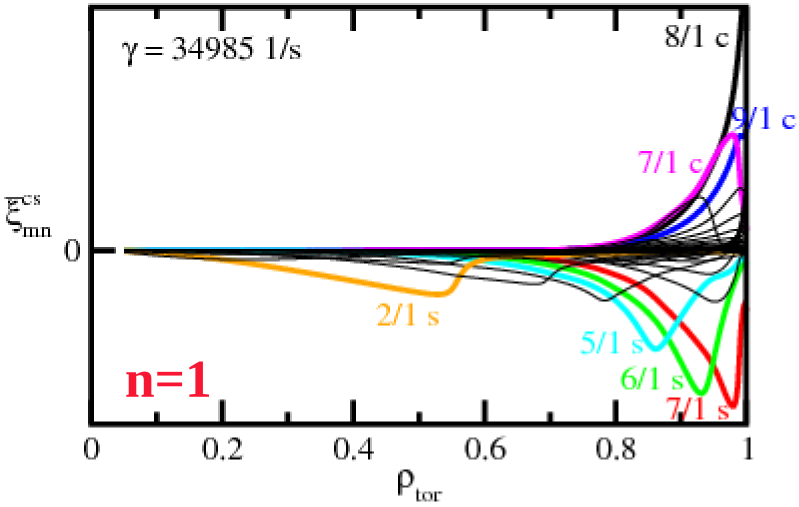

## 3D equilibrium, even case

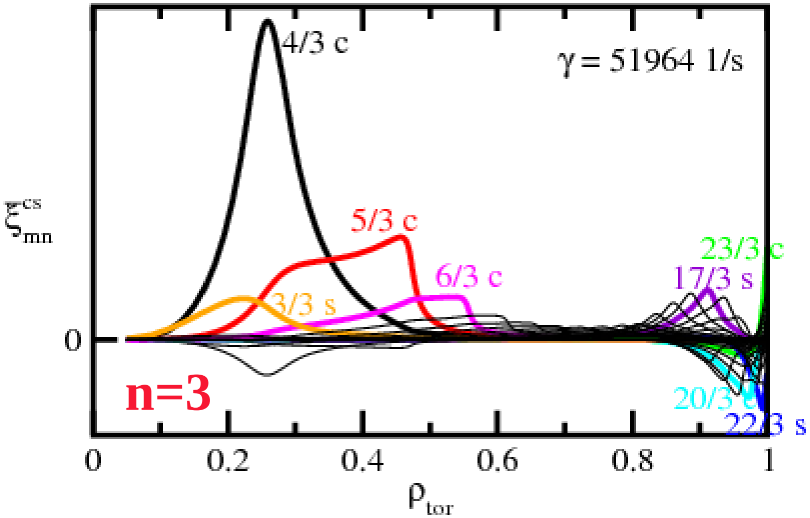


# Stability properties (n=1, 3)

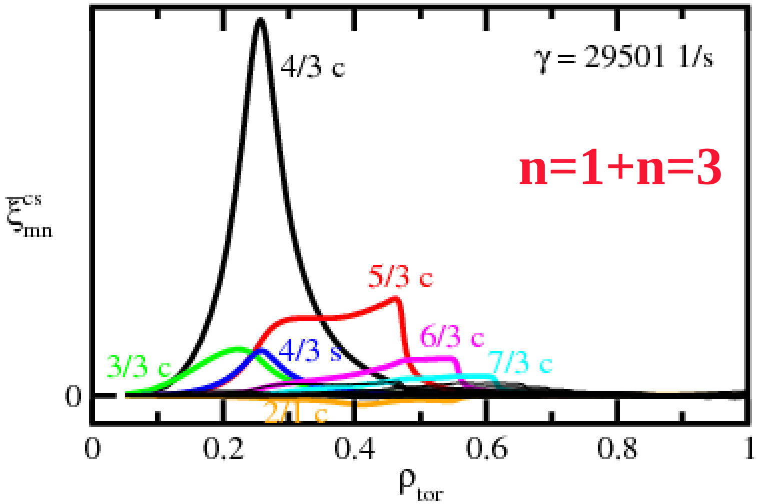
**axisym. equilibrium**




**3D equilibrium, odd case**

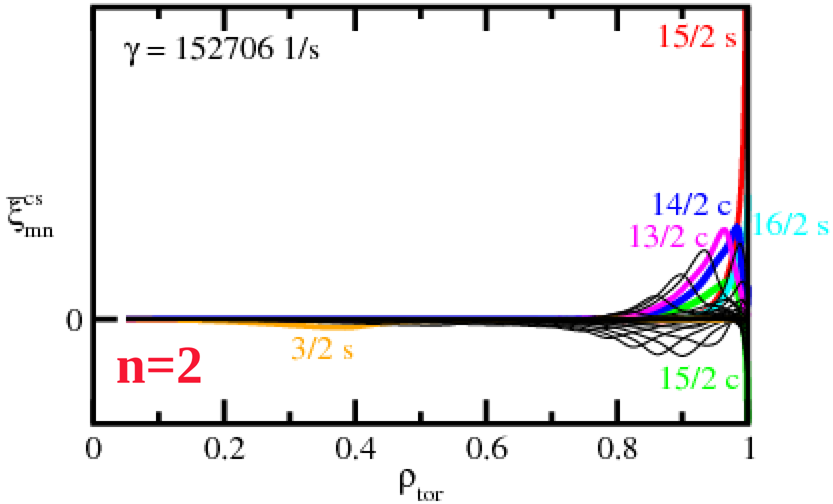
**no unstable n=1 + n=3 external kink mode**

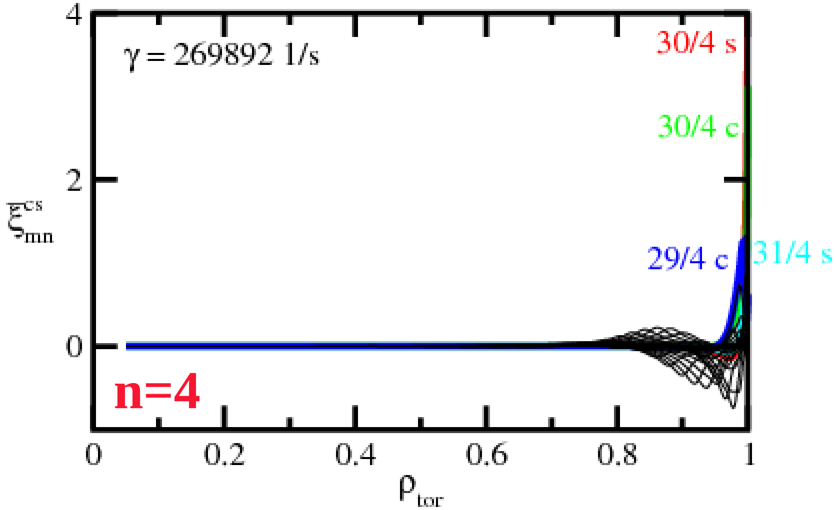


# Stability properties (n=2, 4)

**axisym. equilibrium**

**3D equilibrium**




**In both (even, odd) cases no unstable n=2 + n=4 modes could be found.**

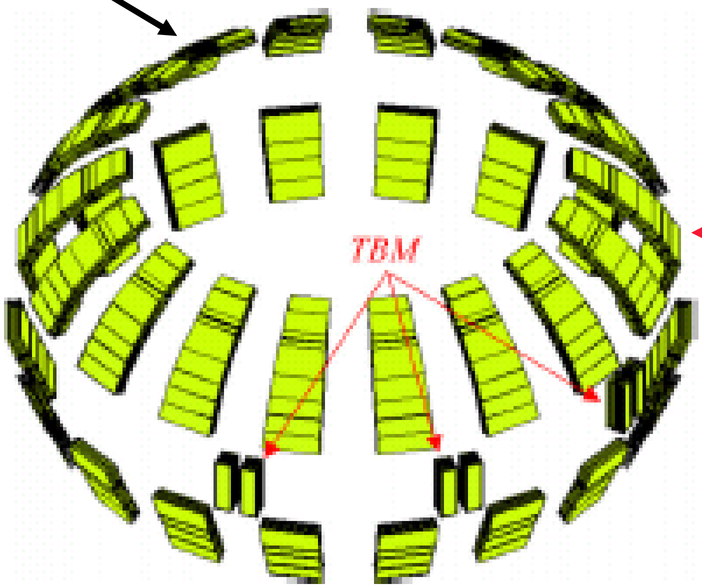
# 3D effects of RMP-coils in ASDEX Upgrade: results

- **The corrugation patterns of the flux surfaces reflect the kink structure of the nearest rational q-surfaces and the periodicity of the perturbation field.**
- **Odd and even RMP-coil currents modulate strength and phase of the corrugation.**
- **Here, both, stabilizing and destabilizing effects of the RMP-field have been found.**
- **In order to get an accurate eigenvalue many poloidal harmonics (here: ~ 30 poloidal harmonics per n) and several n have to be taken into account. The two toroidal harmonics, which have been considered here, are very probably not enough.**

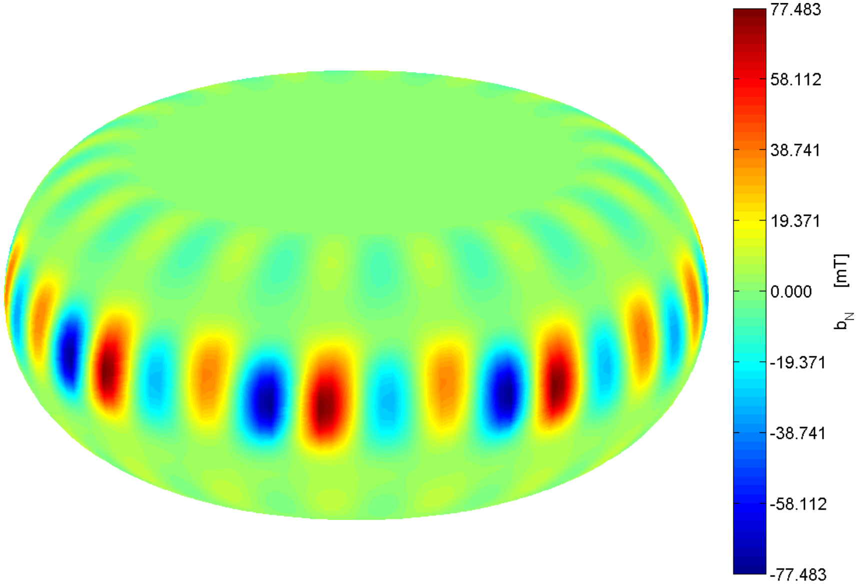
→ **a tremendous numerical effort is necessary**

# Test Blanket Modules (TBMs) in ITER

K. Shinohara et al., Nucl. Fusion 51 (2011) 063028

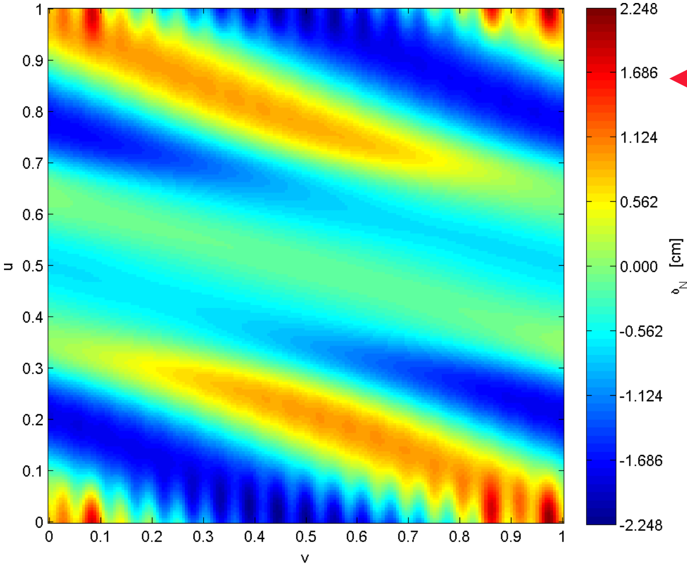



- The finite number of TF-coils (18), ferritic inserts and TBMs cause a magnetic field ripple in ITER.



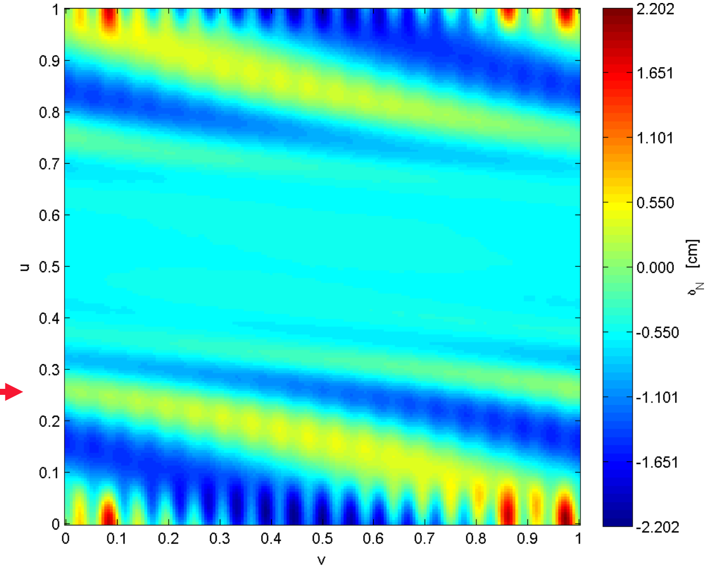

- ITER, $I_p = 9$ MA, $\langle\beta\rangle = 3$ %
- $(b_N/B_0)_{max} = \pm 0.016$
  $b_N$ = normal component of the ripple field

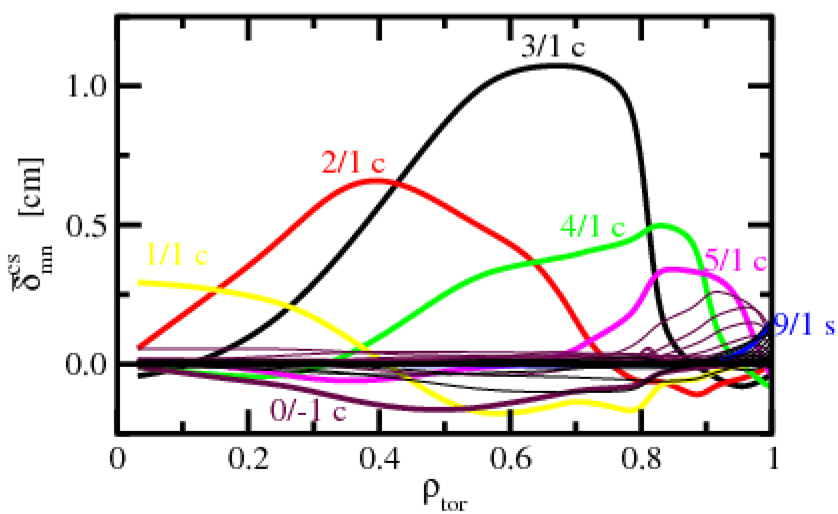
# Corrugation of the flux surfaces




q=3, $\rho_{tor}$= 0.81

q=6.3, $\rho_{tor}$=1

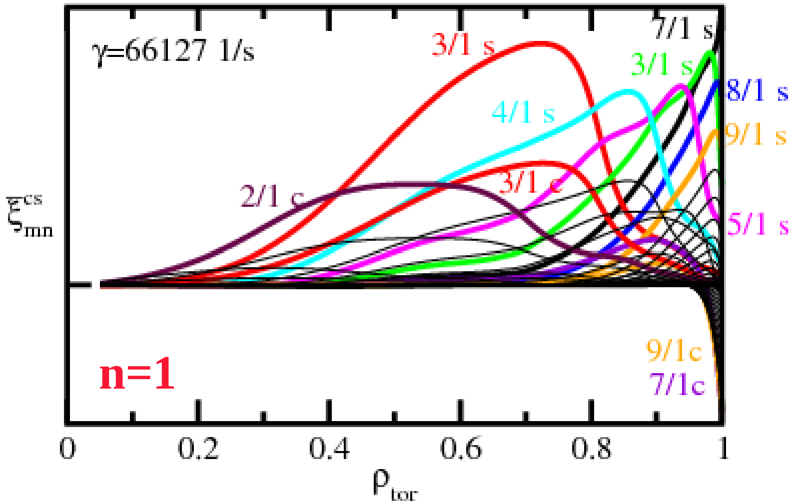


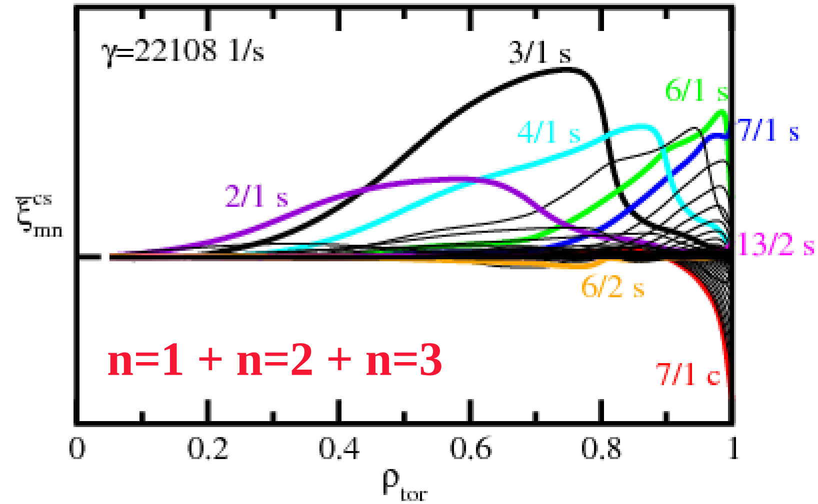

- The n=1 perturbation caused by the TBMs leads to a n=1 kink type bending of the flux surfaces with m=3 being the dominating Fourier harmonic in the interior.
- Superimposed is a n=9 and n=18 dominated corrugation caused by TBMs and TF-ripple.

# Stability properties (n=1,2,3)

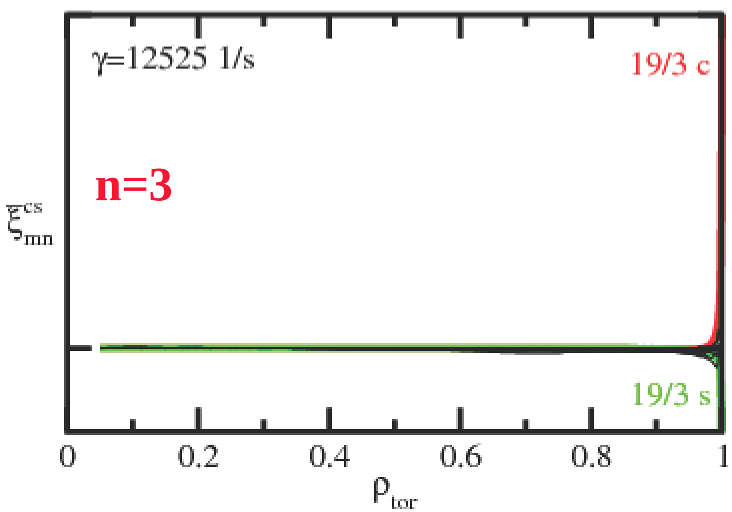
## axisym. equilibrium

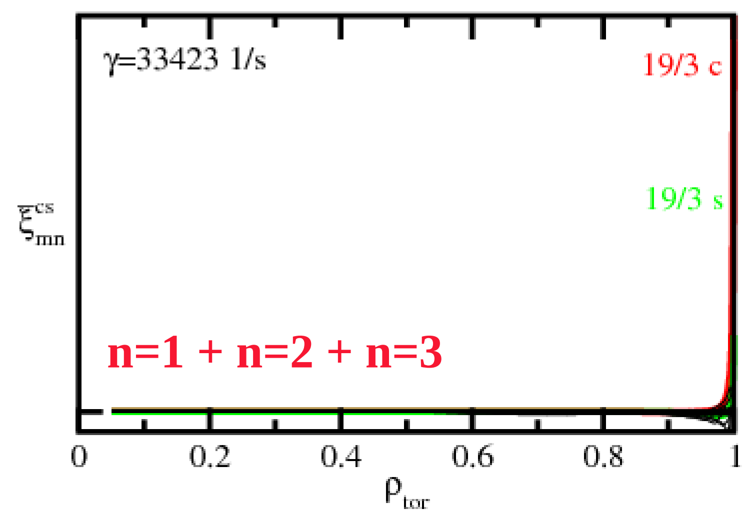



## 3D equilibrium




n=2 stable



# 3D effects of TBMs in ITER : results

- **eigenvalues [1/s]**

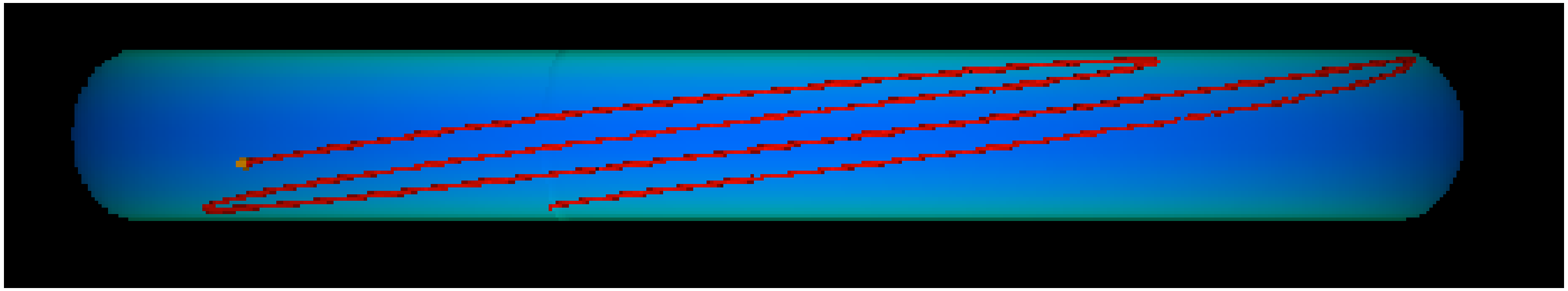
| axisym. | | | 3D equilibrium | | | | |
|---|---|---|---|---|---|---|---|
| n=1 | n=2 | n=3 | n=1 | n=2 | n=3 | n=1,3 | n=1,2,3 |
| 66127 | stable | 12526 | stable | stable | 33129 | 33283 | 33423 |
| | | | | | | 33253 | 33411 |
| | | | | | | | 22108 |
| | | | | | | | 17741 |

→ **The coupling of the toroidal harmonics plays an important role.**

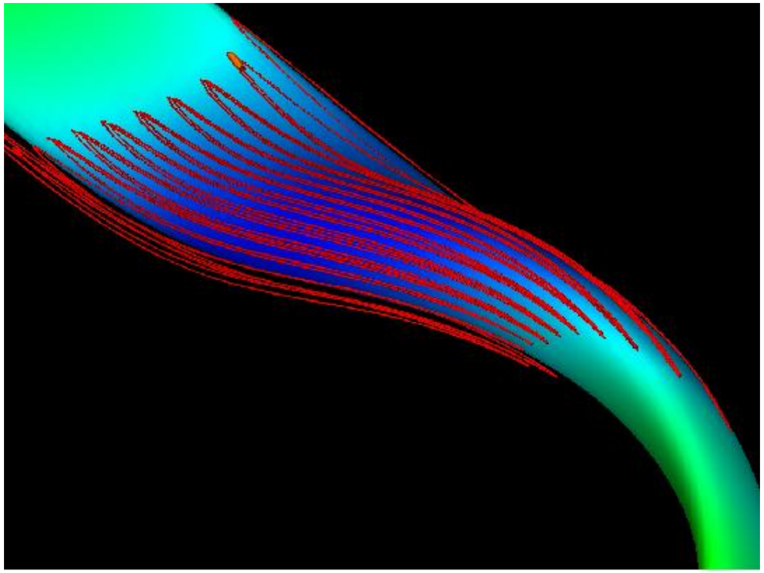
- **The TBMs can influence the stability properties.**

# Confinement of trapped particles

**axisymmetric tokamak**



**isodynamic stellarator (W7-X)**

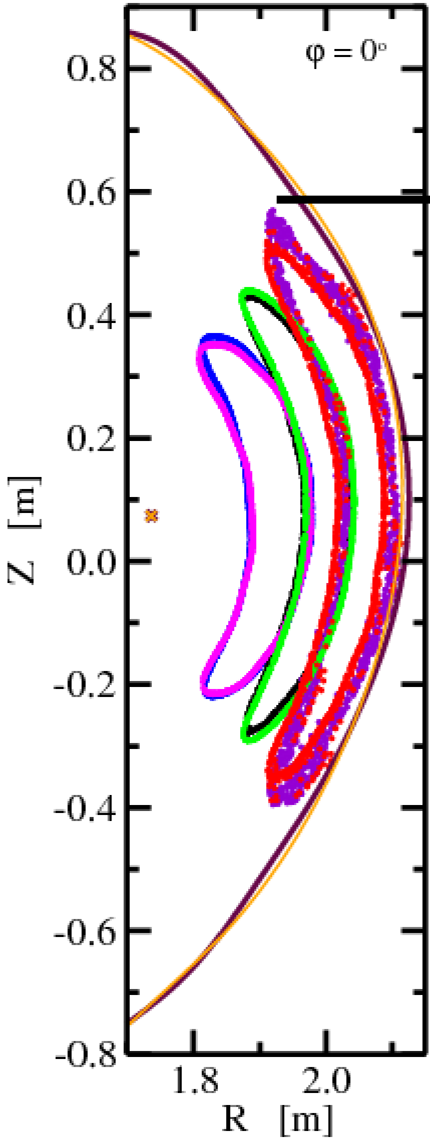


- **A trapped particle is confined if the contour of its second adiabatic invariant**

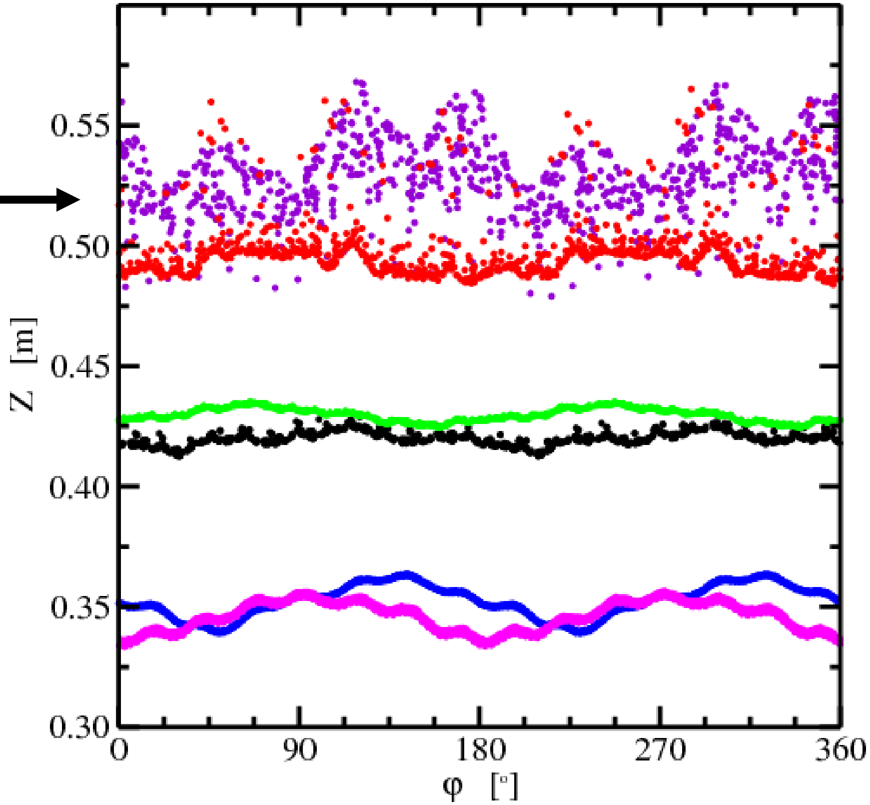
$$\mathcal{J} = \int v_{\parallel}\, dl$$

**is closed. That is, the drift orbit of the guiding centre is closed.**

# Confinement of trapped particles (cont'd)




upper reflection points




- 3D AUG equilibria (odd and even cases, $N_P$= 2)
- guiding centres of 93 keV Deuterons traced for 0.02 s

colours: odd (blue, black, violet, maroon)
even (magenta, green, red, orange)

# Confinement considerations: results

- In 3D tokamaks the drift orbits of trapped particles are not closed.

  → deterioration of the particle confinement

- The positions of the reflection points reflect:

  - the periodicity of the perturbation, and
  - the phase shift between odd and even perturbation fields

# Acknowledgement

**I would like to thank Carolin Nuehrenberg for many important hints and explanations concerning the use of the CAS3D code, and Peter Merkel for many useful discussions.**