MHD instabilities in tokamaks with 3D effects

E. Strumberger,

S. Guenter, and C. Tichmann

531. WE-Heraeus-Seminar
Bad Honnef, May 2013
OUTLINE

- Introduction
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- Stellarator versus 3D tokamak
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  - 3D effects of RMP-coils in ASDEX Upgrade
  - 3D effects of Test Blanket Modules (TBMs) in ITER

- Confinement considerations
3D effects in tokamaks

- **Toroidal field coils**: mostly high-n perturbations, e.g. \( n=16 \) in AUG
- **Test blanket modules**: low-n perturbation \((n=1)\) in ITER
- **RMP coils**: low-n perturbations, e.g. \( n=1, 2 \) or 4 in AUG
- **Resistive wall**: e.g. medium-n perturbation \((n=9)\) in ITER
- **Equilibrium with helical core**: low-n perturbation, observed in MAST, TCV, RFX, ....
- **Error fields**: small undesignedly or unavoidable non-axisymmetric magnetic fields \((\Delta B/B) \sim 10^{-4}\)

\( n = \) leading toroidal harmonic of the magn. field perturbation
# 3D equilibrium and stability codes

## 3D equilibrium codes

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEMEC</td>
<td>upgraded version of NEMEC=NESTOR(^1)+VMEC(^2) code, P. Merkel, S. Hirshman, 3D free-bound. equilib. (assump. of nested flux surf.)</td>
</tr>
<tr>
<td>ANIMEC</td>
<td>W. A. Cooper, variant of the VMEC code designed to obtain 3D anisotropic pressure equilibria</td>
</tr>
<tr>
<td>PIES</td>
<td>A. Reiman, D. Monticello, 3D equilibrium code, handles islands and stochastic regions</td>
</tr>
<tr>
<td>HINT</td>
<td>T. Hayashi, 3D equilibrium code, handles islands and stochastic regions</td>
</tr>
</tbody>
</table>

## Coordinate transformation into Boozer coordinates

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>COTRANS</td>
<td>E. Strumberger, coordinate transformation and code interface, contains parts of the JMC code of J. Nuehrenberg and R. Zille</td>
</tr>
</tbody>
</table>

## 3D linear ideal stability codes

<table>
<thead>
<tr>
<th>Code</th>
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</tr>
</thead>
<tbody>
<tr>
<td>CAS3DN</td>
<td>modified version (Non-equidistant radial grid) of the CAS3D code of C. Nuehrenberg</td>
</tr>
<tr>
<td>TERPSICHORE</td>
<td>D.V. Anderson, W.A. Cooper, uses finite elements in radial direction and Fourier decomposition in angular variables similar to CAS3D</td>
</tr>
</tbody>
</table>
**axisymmetry**

\[ B = B(R, Z) \]

**stellarator symmetry**

\[ B(r') = B(r) \]
\[ B_R(r') = -B_R(r), \quad B_\phi(r') = B_\phi(r), \quad B_Z(r') = B_Z(r) \]

with \( r = (R, \phi, Z) \), \( r' = (R, 2\pi/N_p - \phi, -Z) \)

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**up-down symmetric**

**not stellarator symmetric**

**one period**
Field periods and mode families

W7-X, 5 field periods ($N_p=5$)

- Depending on the perturbation, 3D tokamaks may have $N_p=1, 2, 3, \ldots$

- Because of the periodicity only those toroidal harmonics contribute to a perturbation which differ by multiples of $N_p$. Such a group is called mode family.

- Depending on $N_p$, there are $1+[N_p/2]$ mode families.

_axisym._ configuration: $N_p = \infty$, all n’s decouple

_3D configuration: $N_p = 1$, all n’s couple

$N_p = 2$, two mode families, $n=0, 2, 4, \ldots$ and $n=1, 3, 5, \ldots$

$N_p = 5$, three mode families, $n=0, 5, 10, 15, \ldots$

$n=1, 4, 6, 9, 11, \ldots$

$n=2, 3, 7, 8, 12, 13, \ldots$
The safety factor profile of a tokamak contains much more rational surfaces than that of a stellarator.

The stability computations for 3D tokamak configurations require a much higher numerical effort with respect to:

- resolution of the radial grid
- number of poloidal and toroidal Fourier harmonics which describe the displacement vector
Magnetic coordinates (Boozer coordinates)

- The poloidal coordinate lines in a stellarator are smooth lines with small curvature.

- In a tokamak the poloidal coordinate lines become almost parallel to the radial ones for high q-values.

- Their corrugation increases with increasing 3D deformation of the flux surfaces.
Stellarator versus 3D tokamak: results

The numerical effort for
- transformation into magnetic coordinates, and
- stability studies
is much higher for 3D tokamak equilibria than for stellarator equilibria, because of
- the numerous rational surfaces, and
- the strong bending of the poloidal coordinate lines.
The **NEMEC**[^1] (NEstor+VMEC) code computes fixed- and free-boundary, low- and high-beta, axisymmetric equilibria with an excellent numerical accuracy ($\text{ftol} < 10^{-11}$).

The stability properties of the equilibria play no role, because the computations are restricted to axisymmetry.

Scaling the pressure profile, the plasma beta has been increased. Simultaneously, plasma shape and boundary have been kept fixed roughly by a suitable choice of the coil currents (free-boundary cases).

A helical core with a 3/2 geometry develops.

Comparable results have been found previously, e.g.:
- W.A. Cooper et al., PRL 105 (2010) 035003
  "The helical equilibrium states resemble saturated internal kink mode structures."
Analysis of the 3D fixed-boundary equilibrium solution

**measure of the corrugation:**

\[ \vec{\delta}_N = \delta_N \vec{n} \]

\( \vec{\delta}_N \): vector pointing in normal direction from the average axisymmetric flux surface to its corresponding 3D surface

\( \delta_N \): corresponds to the normal component of the displacement vector in linear MHD stability computations

\( |\delta_N|_{\text{max}}(s_i) \): max. absolute value of flux surface \( i \)

\( |\delta_N|^{e}_{\text{max}} \): max. absolute value of the equilibrium

Fourier representation of \( \delta_N \) in magnetic coordinates (Boozer coor.):

\[
\delta_N(s_i) = \sum_{m=0, n=-n_b}^{m_b, n_b} \bar{\delta}_{m,n}^c(s_i) \cos(m\theta + n\phi) + \bar{\delta}_{m,n}^s(s_i) \sin(m\theta + n\phi)
\]
Analysis of the 3D fixed-bound. equilib. solution (cont’d)

Equilibrium: \[ \delta W_{MHD} = 0 \quad \iff \quad \vec{F} = -\vec{j} \times \vec{B} + \nabla p = 0 \]

Energy difference between axisymmetric and 3D equilibrium:

\[ \Delta W_{MHD} \sim 300 \text{ J} \]
The Fourier spectrum of $\delta_N$ shows the structure of an internal kink mode. With increasing 3D deformation more and more toroidal harmonics, $n$, are involved. Since the equilibrium computation has been performed for a two periodic configuration, only toroidal harmonics $n=0,2,4,…$ appear.
3D equilibrium calculations: results

Equilibria with a helical core and an axisymmetric boundary can be found with 3D NEMEC fixed-boundary computations, if the corresponding axisymmetric equilibrium is unstable with respect to internal kink modes.

The corrugation of the flux surfaces reflects the ideal kink structure.

The energy difference between the axisymmetric and the 3D equilibrium state is very small. Here:

\[
\frac{\Delta W_{\text{MHD}}}{W_{\text{MHD}}} \lesssim 10^{-5}
\]

3D equilibrium solutions are very sensitive to numerical parameters and accuracy. Those affect the corrugation and, therefore, also the succeeding stability and confinement studies.

3D equilibrium calculations are very demanding.
The chosen number of periods for the equilibrium calculation selects the type of the helical core, e.g. $m=1$, $n=7$ for $N_p=7$ in RFX-mod, or $m=3$, $n=2$ for $N_p=2$ in the considered test case.

Assuming a fixed-boundary also restricts the 3D equilibrium solution.

Poorly, or non-converging solutions indicate that in such cases no 3D equilibrium exists.

The 3D deformation of the plasma core has a stabilizing effect on the considered pure \( n=1 \) internal kink mode. For a large enough core the mode becomes eventually stabilized.
Stability studies: equilibrium with helical core (cont’d)  
(fixed-boundary)

With increasing 3D deformation of the flux surfaces the coupling of the toroidal harmonics become more and more important.
Here the 3/2 helical core has a greater stabilizing effect on the even than on the odd mode family.
The resulting 3D fixed-boundary equilibrium (3/2 helical core structure) of the considered test case is still internal unstable with respect to
- $n=3$ with a considerable part of $n=1$ harmonics,
- $n=4$ with a considerable part of $n=2$ harmonics, and
- most likely higher toroidal harmonics.
This is most likely the main reason for the moderate convergence of the equilibrium solution and its final divergence.

The quality of a 3D equilibrium solution and its stability properties are strongly correlated.
RMP coils in AUG

\( n=2 \) perturbation field

even

odd

\( (b_N/B_0)_{\text{max}} = \pm 0.004 \)

\( b_N = \) normal component of the perturbation field

\( \text{AUG-type equilibrium, } <\beta> = 2\% \)
Corrugation of the flux surfaces

- q~15/2, $\rho_{\text{tor}}=1$
- The corrugation of the flux surfaces is larger for the odd case in the boundary region, but almost the same in the core region.
- There is a shift shift between odd and even case.

q=3/2, $\rho_{\text{tor}} = 0.4$
Corrugation of the flux surfaces (cont’d)

Fourier spectra (mag. coordinates)

even

odd

maximum corrugation as function of $\rho_{\text{tor}}$
Stability properties (n=1, 3)

axisym. equilibrium

3D equilibrium, even case

n=1

n=3

n=1 + n=3

n=1 + n=3
Stability properties (n=1, 3)

axisym. equilibrium

3D equilibrium, odd case

no unstable n=1 + n=3
external kink mode
Stability properties ($n=2$, $4$)

axisym. equilibrium

3D equilibrium

In both (even, odd) cases no unstable $n=2 + n=4$ modes could be found.
The corrugation patterns of the flux surfaces reflect the kink structure of the nearest rational q-surfaces and the periodicity of the perturbation field.

Odd and even RMP-coil currents modulate strength and phase of the corrugation.

Here, both, stabilizing and destabilizing effects of the RMP-field have been found.

In order to get an accurate eigenvalue many poloidal harmonics (here: ~ 30 poloidal harmonics per n) and several n have to be taken into account. The two toroidal harmonics, which have been considered here, are very probably not enough.

→ a tremendous numerical effort is necessary
Test Blanket Modules (TBMs) in ITER

K. Shinohara et al., Nucl. Fusion 51 (2011) 063028

The finite number of TF-coils (18), ferritic inserts and TBMs cause a magnetic field ripple in ITER.

- ITER, $I_p = 9$ MA, $\langle \beta \rangle = 3\%$
- $(b_N/B_0)_{\text{max}} = \pm 0.016$
- $b_N = \text{normal component of the ripple field}$
Corrugation of the flux surfaces

- The n=1 perturbation caused by the TBM s leads to a n=1 kink type bending of the flux surfaces with m=3 being the dominating Fourier harmonic in the interior.
- Superimposed is a n=9 and n=18 dominated corrugation caused by TBM s and TF-ripple.
Stability properties (n=1,2,3)

axisym. equilibrium

\[ \gamma = 66127 \, \text{1/s} \]

- \( n=1 \)
- \( n=2 \) stable
- \( n=3 \)

3D equilibrium

\[ \gamma = 22108 \, \text{1/s} \]

- \( n=1 + n=2 + n=3 \)

\[ \gamma = 12525 \, \text{1/s} \]

- \( n=1 + n=2 + n=3 \)

\[ \gamma = 33423 \, \text{1/s} \]

- \( n=1 + n=2 + n=3 \)
The coupling of the toroidal harmonics plays an important role.

The TBMs can influence the stability properties.
A trapped particle is confined if the contour of its second adiabatic invariant

\[ J = \int v_i \, dl \]

is closed. That is, the drift orbit of the guiding centre is closed.
Confinement of trapped particles (cont’d)

3D AUG equilibria (odd and even cases, $N_p = 2$)
Guiding centres of 93 keV Deuterons traced for 0.02 s

Colours: odd (blue, black, violet, maroon)
Even (magenta, green, red, orange)
Confinement considerations: results

- In 3D tokamaks the drift orbits of trapped particles are not closed.
  - deterioration of the particle confinement

- The positions of the reflection points reflect:
  - the periodicity of the perturbation, and
  - the phase shift between odd and even perturbation fields
I would like to thank Carolin Nuehrenberg for many important hints and explanations concerning the use of the CAS3D code, and Peter Merkel for many useful discussions.