3D tokamak equilibrium states and impact on fast ion confinement

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Outline

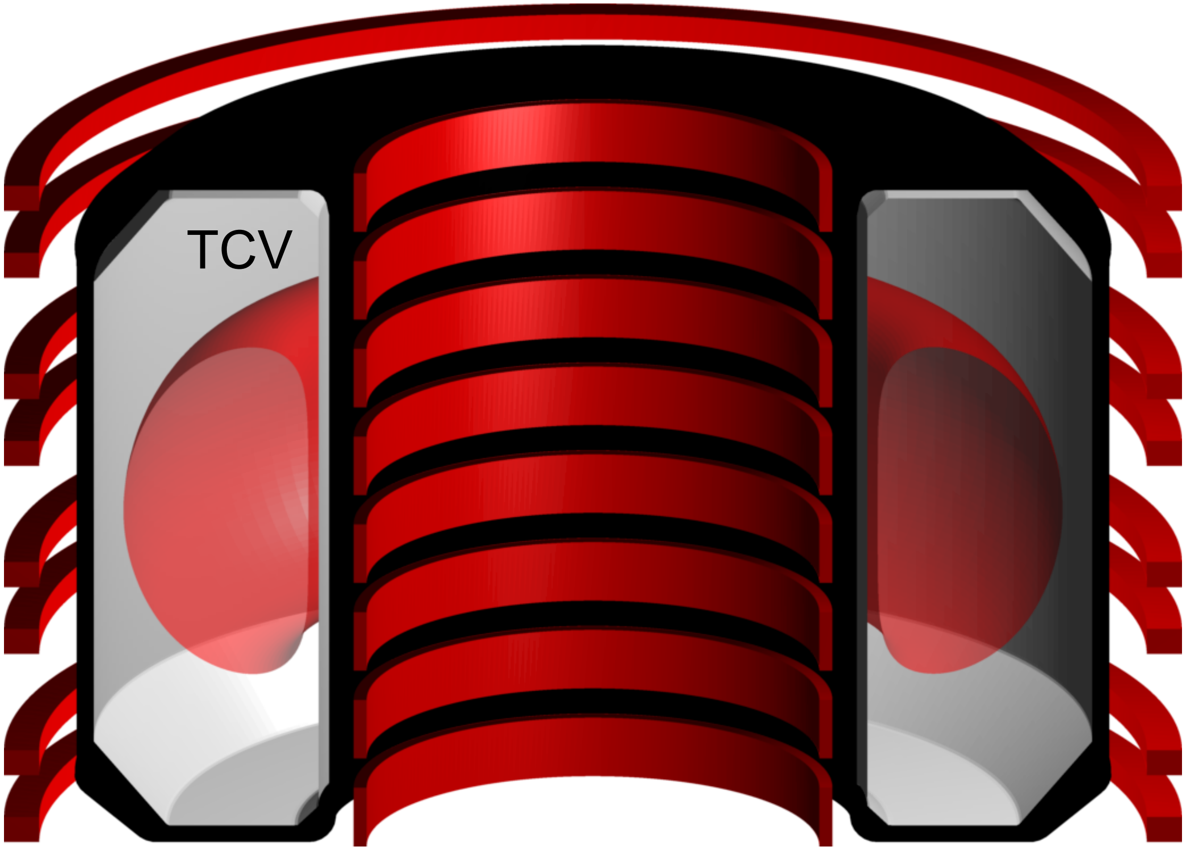
- Helical ideal equilibria in tokamaks: hybrid like scenarions in MAST, TCV and JET.
- Use of 3D equilibrium code for evaluating helical core.
- Comparisons with saturated solutions of nonlinear ideal stability code employed on top of a 2D (axisymmetric) equilibrium code.
- Numerical challenges of following guiding centre orbits in helical equilibria.
- Fast ion confinement in helical equilibria: example NBI confinement during MAST long lived mode.

Theme

(1) We can assume an exactly axisymmetric plasma boundary

(2) We solve for internal flux surfaces in equilibrium: $\rho\cancelto{0}{\frac{dV}{dt}} = \underline{J}\times\underline{B}-\underline{\nabla}P$

- Relax axisymmetry constraint **inside** plasma



Theme

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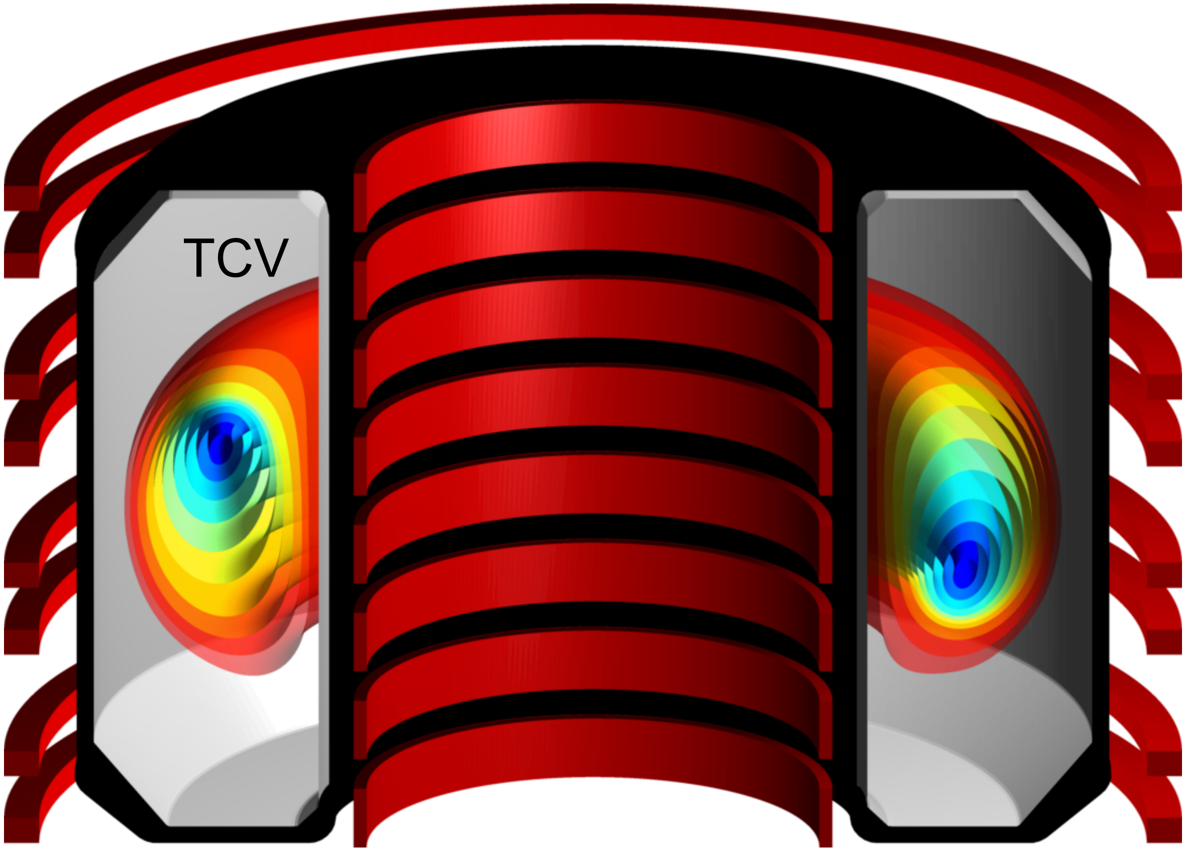
(2) We solve for internal flux surfaces in equilibrium:

$$\cancelto{0}{\rho\frac{dV}{dt}} = \underline{J}\times\underline{B}-\underline{\nabla}P$$

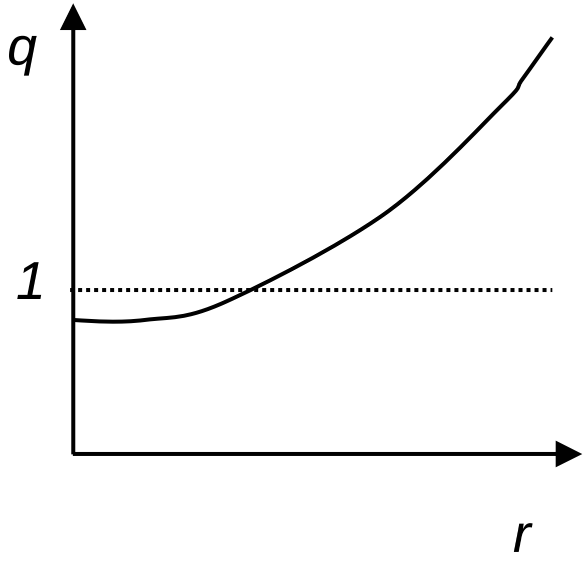
- Relax axisymmetry constraint **inside** plasma

•Two solutions possible:
 - One axisymmetric,
 - the other is helical

•Hybrid scenario susceptible to helical core deformations [Cooper et al, PRL 2010]



Non-linear saturated states



- Rosenbluth [Phys, Fluids **16**, 1984 (1973)] was the first to evaluate the non-linear state of an ideal internal kink (*m=n=1*) displacement.

- This was undertaken for a conventional monotonic *q*-profile. A cylindrical approximation was assumed.

- There were three major conclusions:
  1) A neighbouring kinked equilibrium was found

  2) The non-linear saturated amplitude was “sizable”

  3) The displacement was restricted to the q<1 region

- At the time this was a disappointing result, because an ideal m=n=1 instability could no longer explain rapid disruptions in tokamaks

Tokamak Calculation with monotonic q

- Subsequent numerical tokamak calculations involved initial value simulations to non-linearly propagate ideal MHD from an initially 2D Grad-Shafranov equilibrium.

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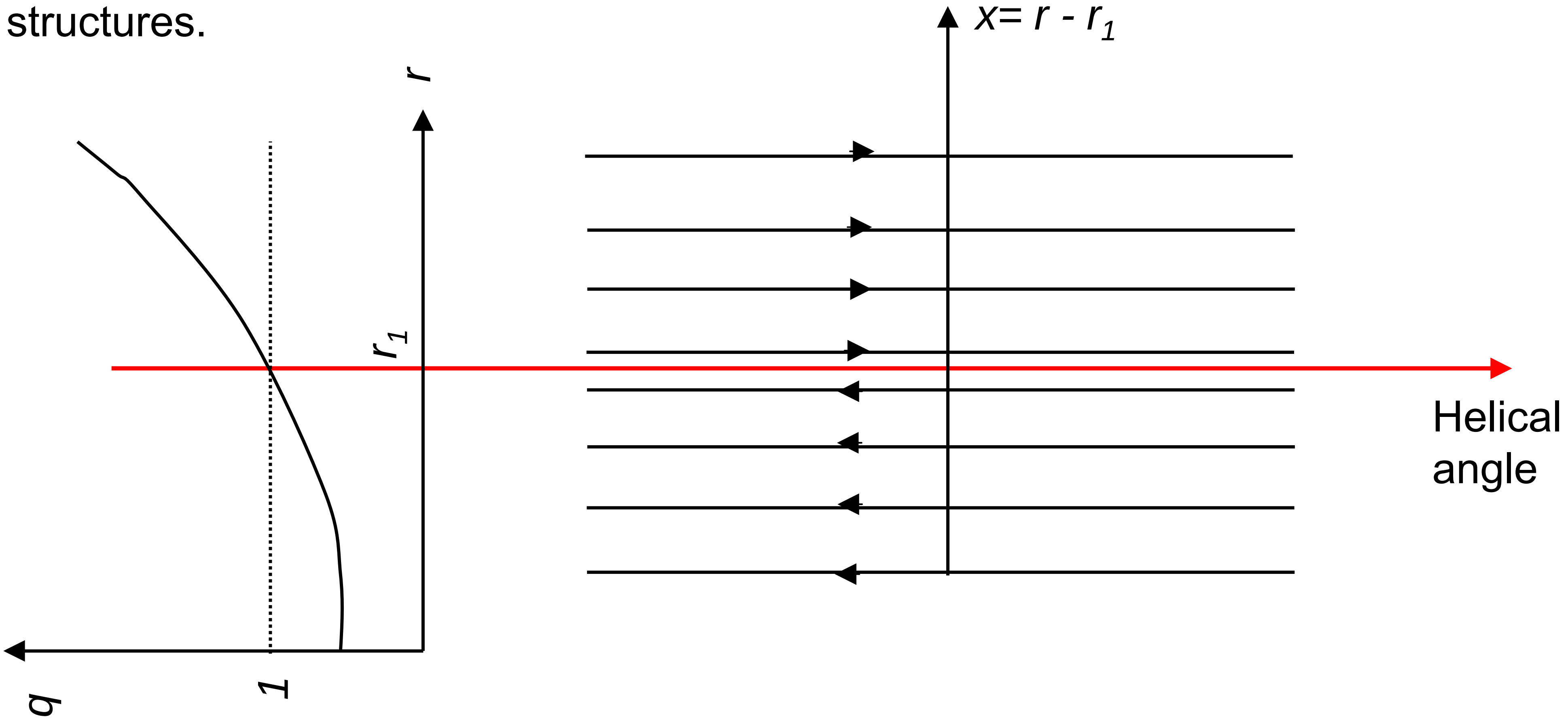

Tokamak Simulations with monotonic q

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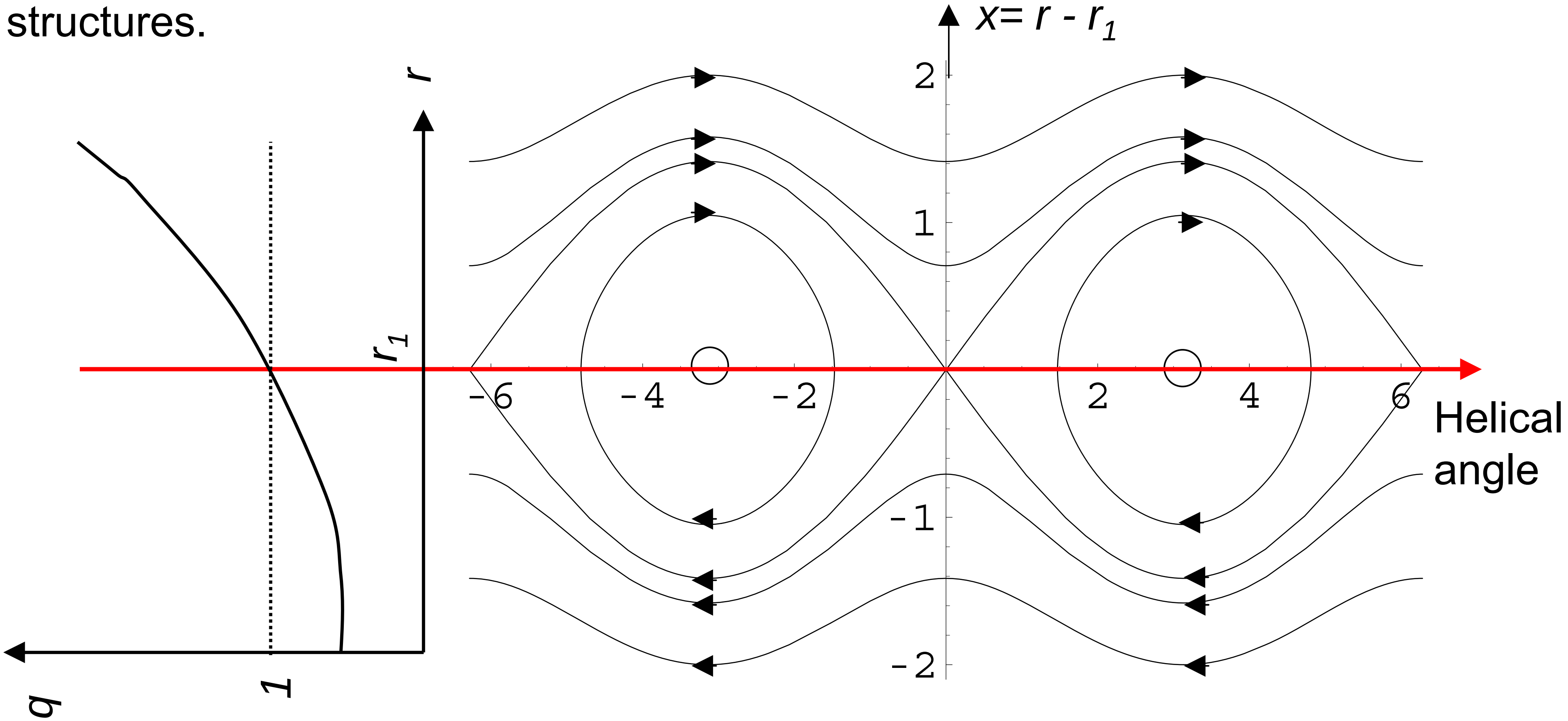
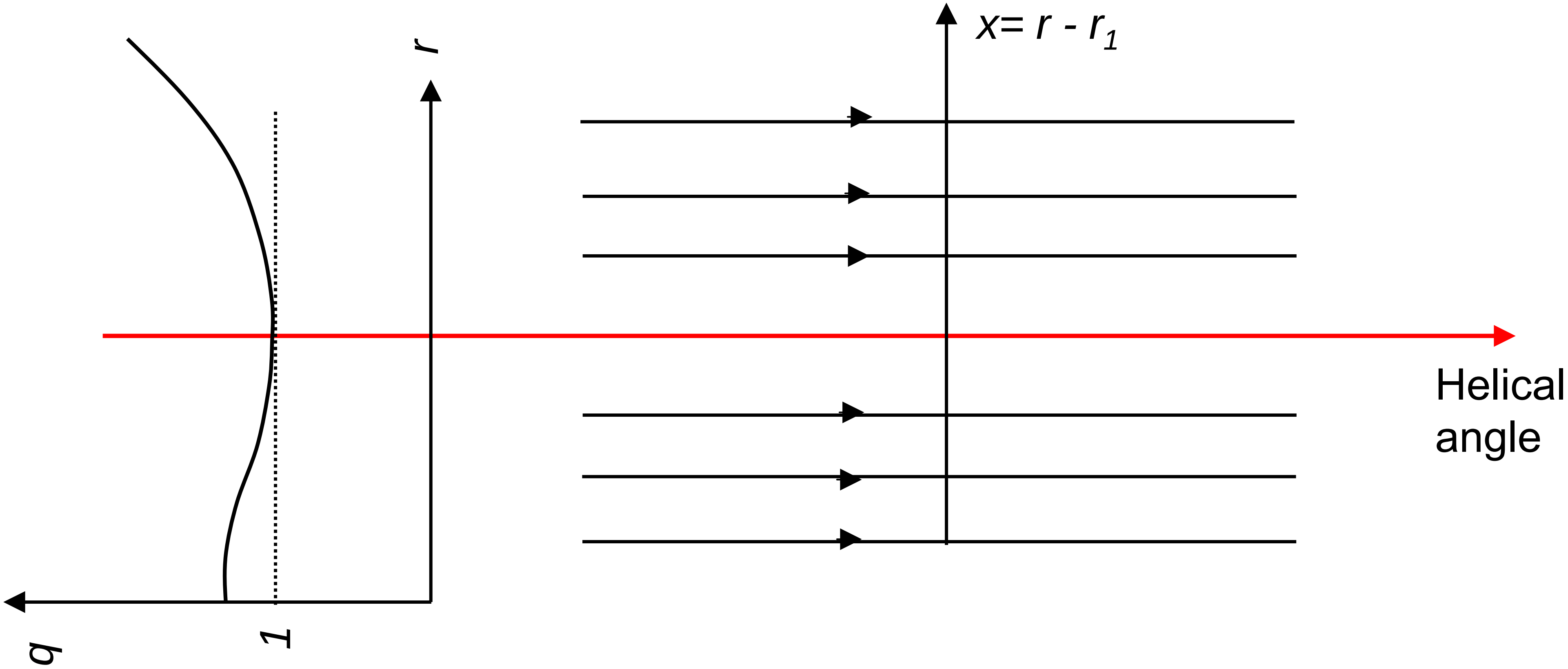


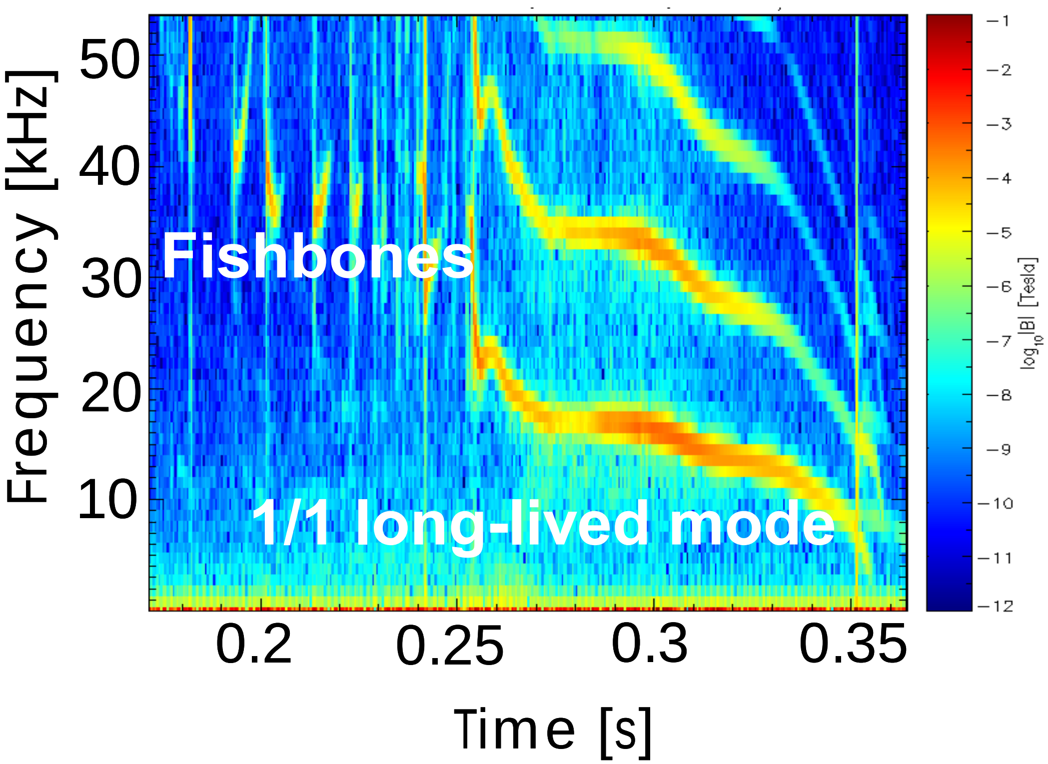
Non-monotonic q profiles

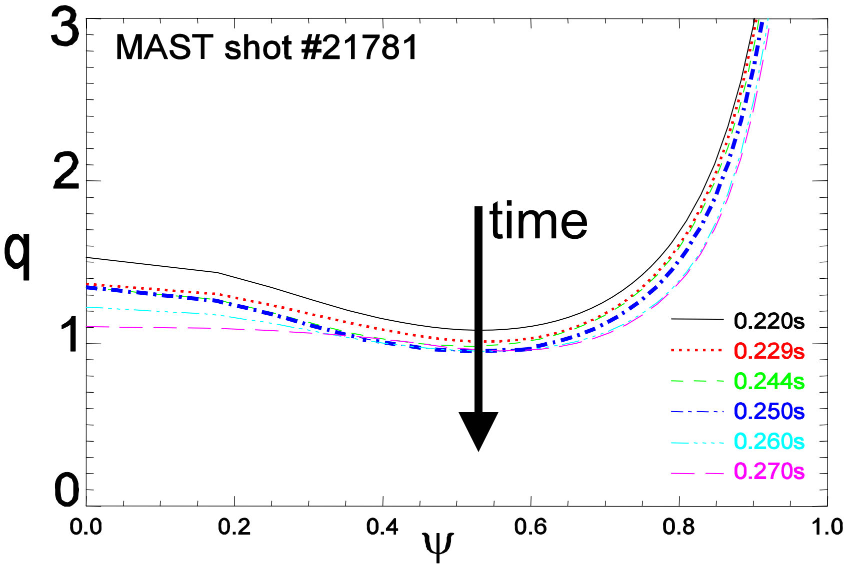
- The non-linear saturated state was considered by Avinash [PRL 59,(1987)] for a non-monotonic q-profile.
- Large n=m=1 helical displacements were calculated for $q_{min}>1$ and $q_{min}=1$.
- Crucially it was noted that such configurations would not be susceptible to magnetic reconnection. Thus an ideal helical state is expected.



Long-Lived Mode (LLM) in MAST

- LLM reported to be [Chapman NF 2010] saturated ideal n=1 mode observed when q-profile is reversed shear or ~flat
- Causes rotation braking and fast ion redistribution

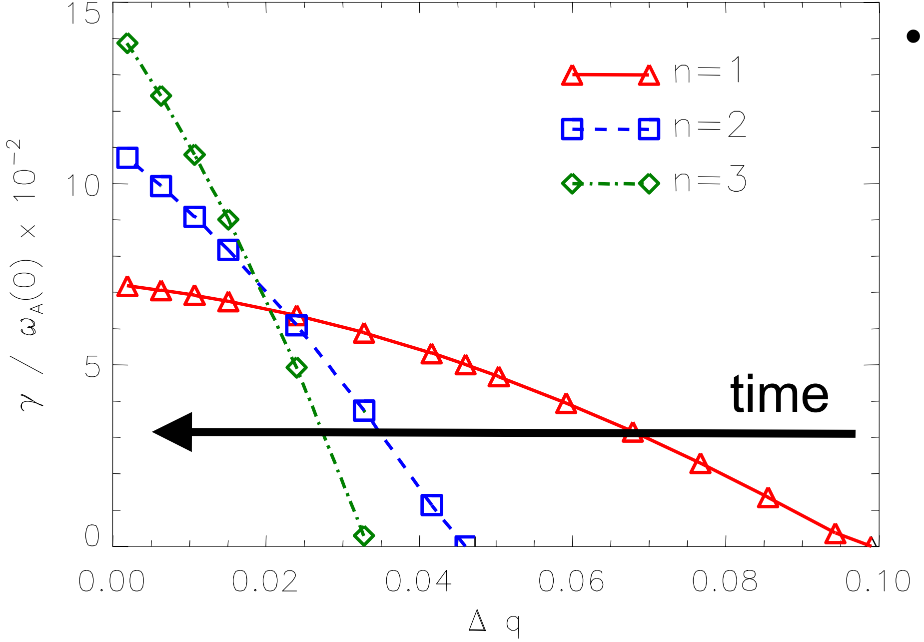


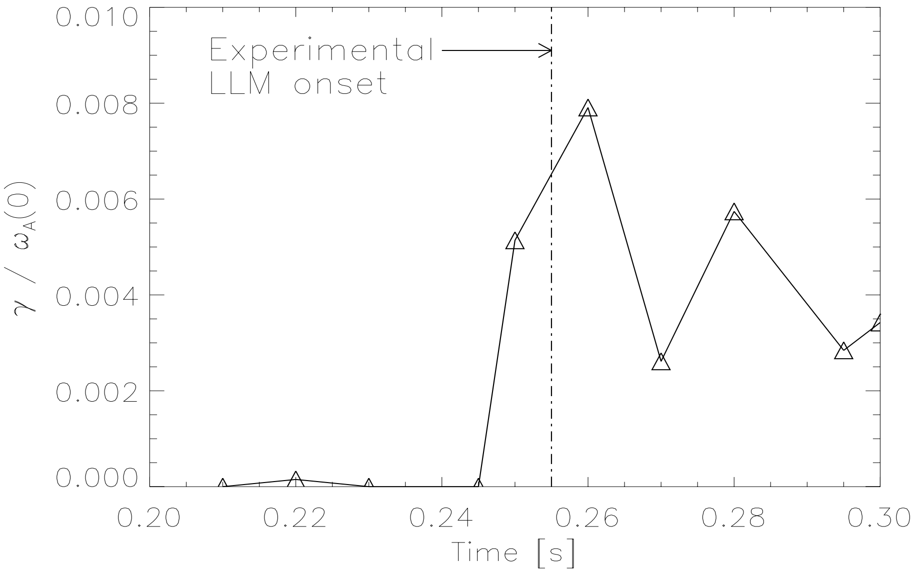


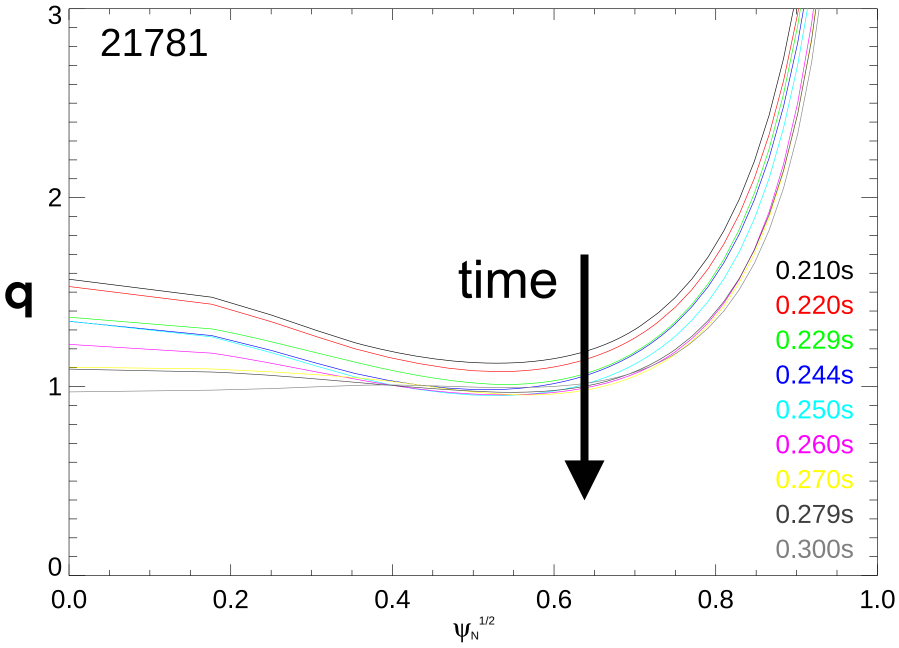
IT Chapman et al, Nucl Fusion, 2010

Ideal Stability in Advanced Tokamak plasmas

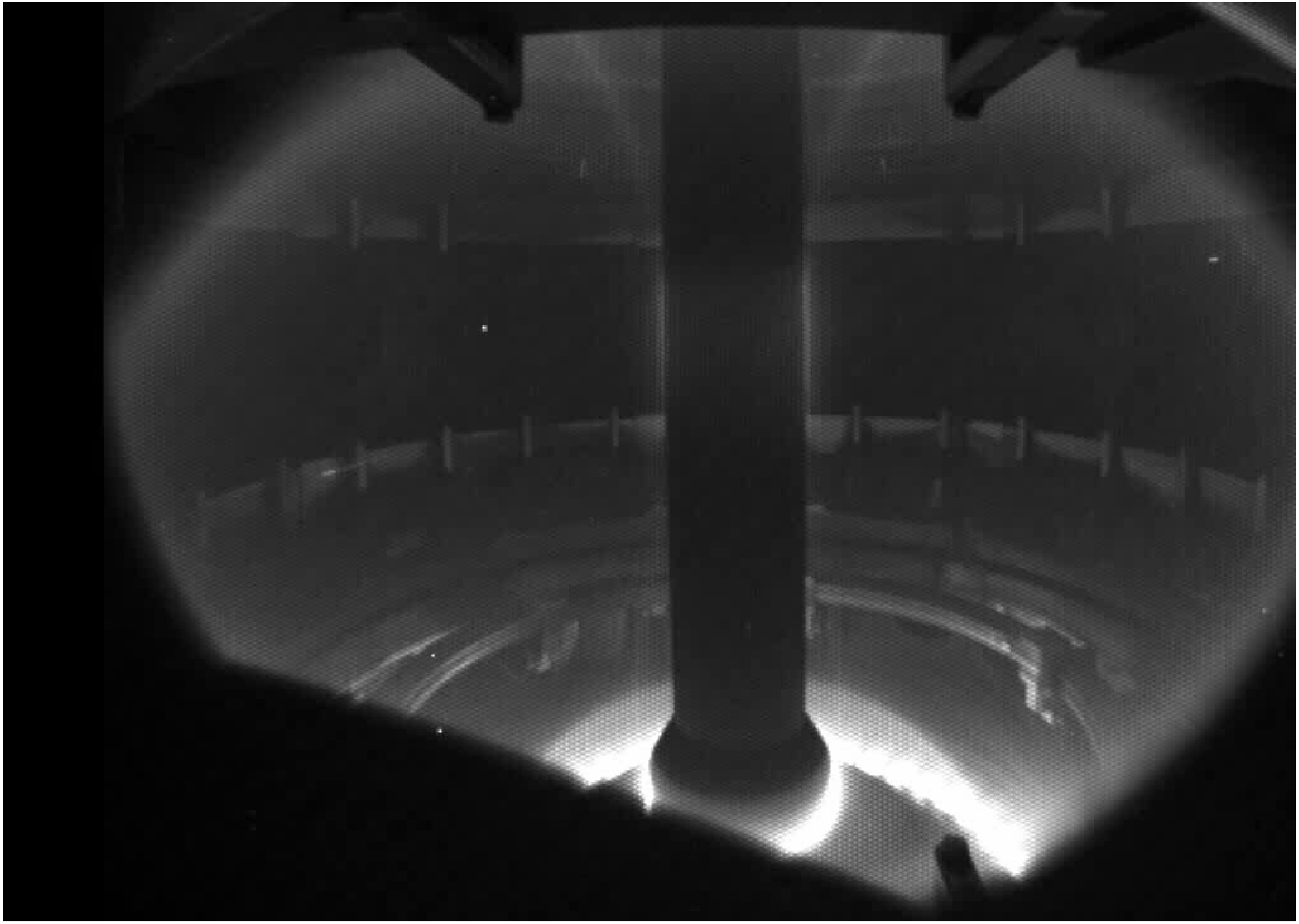
- Plasmas ideally unstable to n=1 internal mode when $q_{min}>1$
  - Hybrid scenario must operate with elevated q-profile to avoid such saturated ideal modes
  - As Δq falls, higher n become more unstable





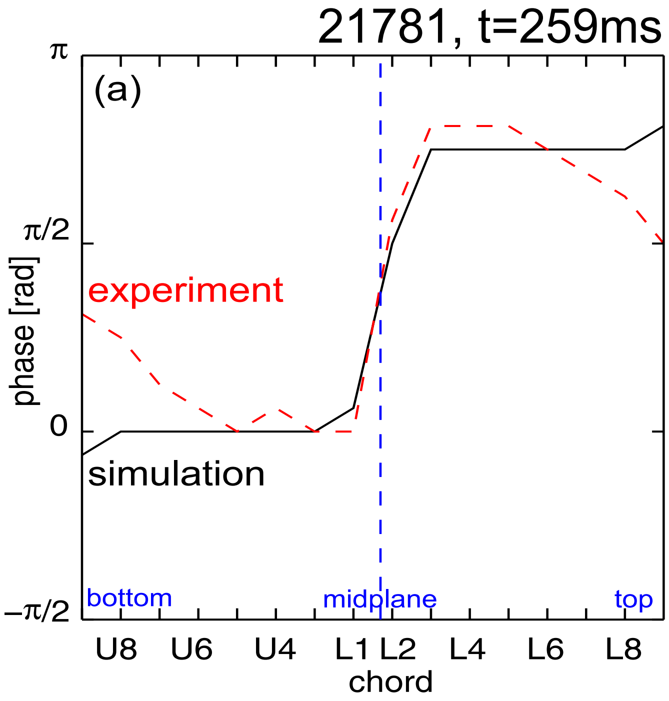


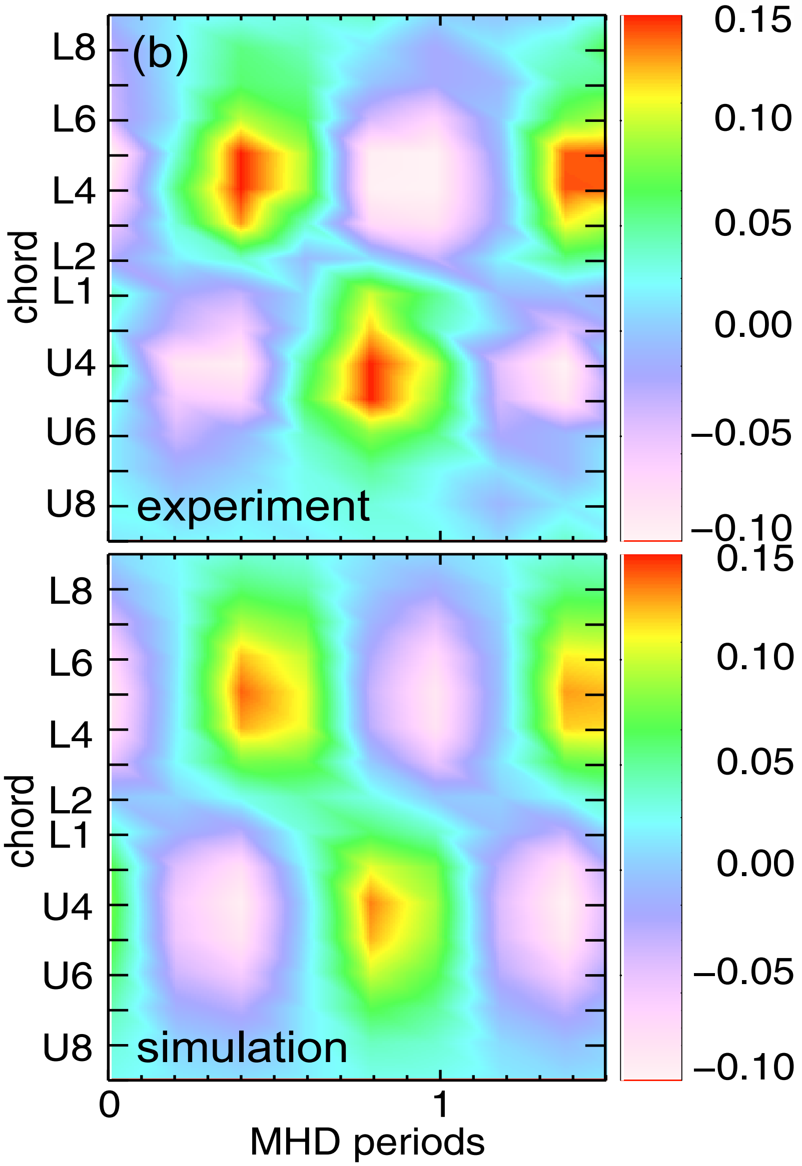
MHD instabilities make tokamaks 3D



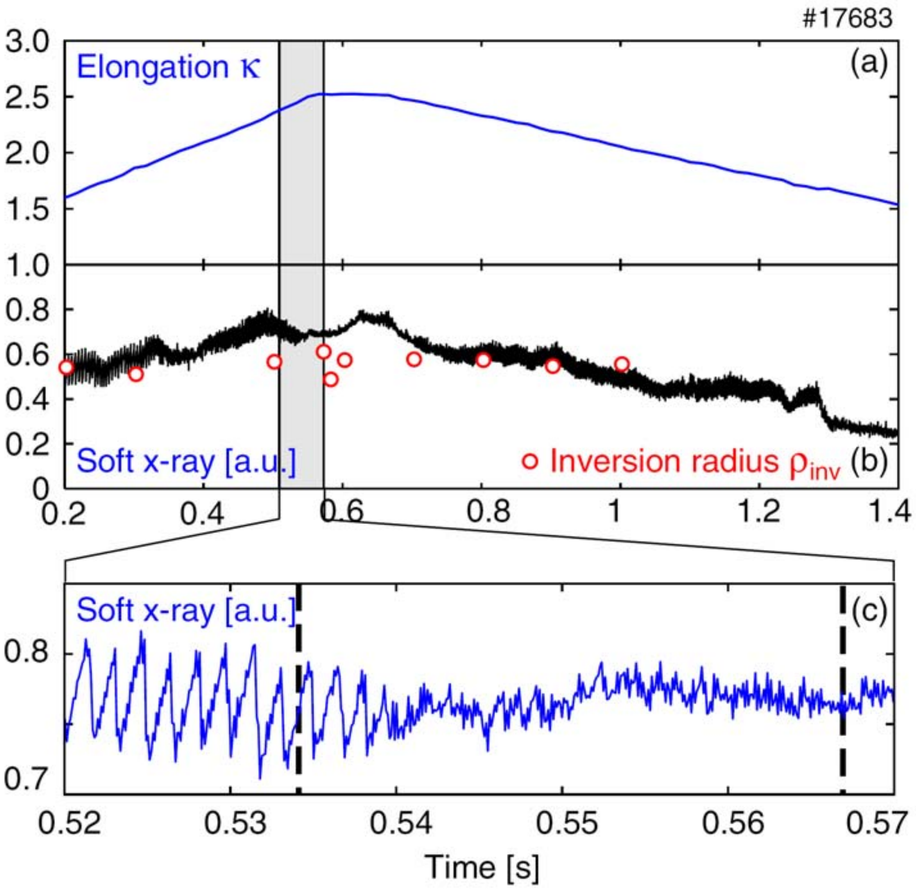
LLM in MAST is an ideal mode

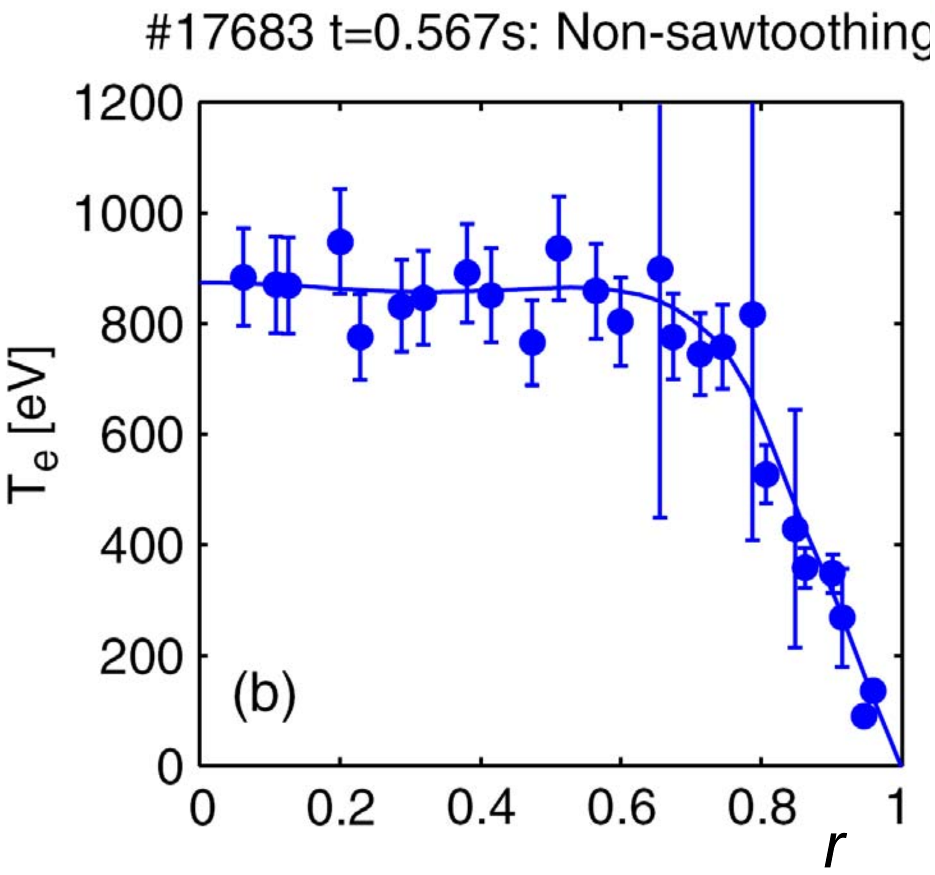
- Amplitude and phase of the Soft X-ray fluctuations agree well with mode structure from linear stability analysis
- No phase jumps observed (typically seen for NTMs)





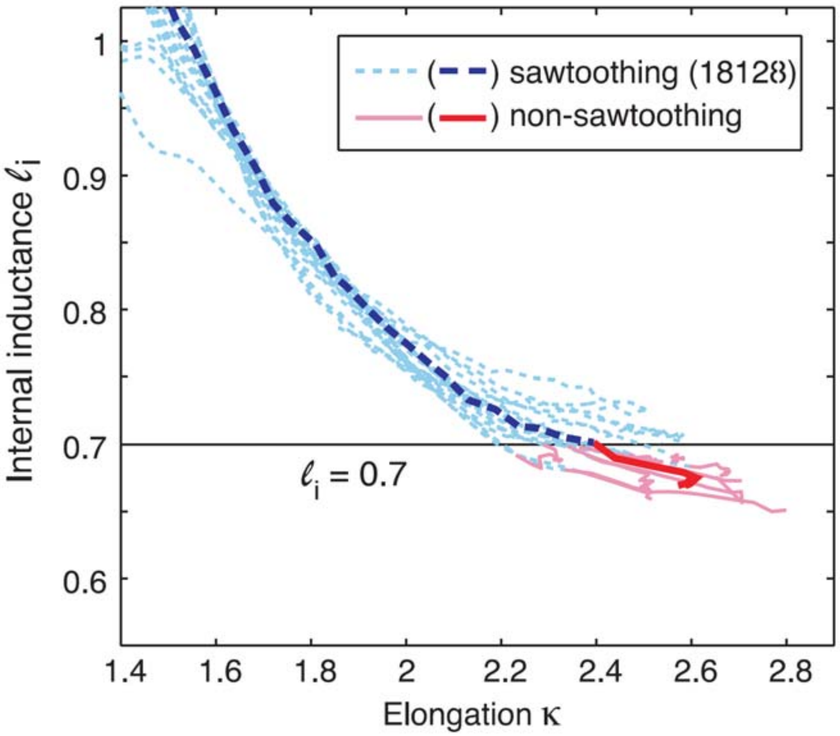
Continuous MHD Oscillations in TCV



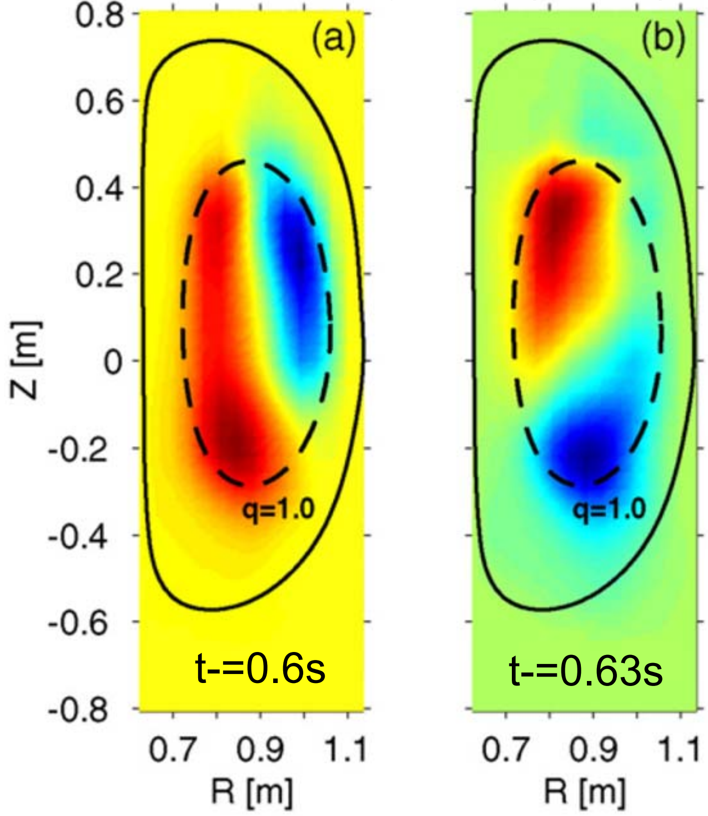


- Increasing the elongation can replaces sawteeth with continuous MHD modes [Reimerdes, PPCF 2006].
- The non-sawtoothing phase is characterised by `continuously' flattened temperature and density profiles.

Continuous MHD Oscillations in TCV



Soft-X tomography (18121)



- •Internal inductance is decreased, leading to the conjecture that q>1 led to this behaviour.
- •Magnetics and soft-x tomography show that mode is dominantly n=m=1. In addition, n=m=2, n=m=3 also observed. This is more evidence of flat q-profile.

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

MHD 3D Equilibrium modelling

CRPP

- Impose nested magnetic flux surfaces and single magnetic axis.
  - Key thing here is to try different guesses for magnetic axis.

- Minimise the energy of the system: $W = \int\int\int d^3x \left(\frac{B^2}{2\mu_0} + \frac{P_{\|}(r,B)}{\Gamma - 1} \right)$

- Solve the inverse equilibrium problem: $R = R(r,\theta,\phi)\ ,\ \ Z = Z(r,\theta,\phi)$

- Variation of the energy:

$$\begin{aligned}\frac{dW}{dt} = \; & - \int\int\int dr\, d\theta\, d\phi \left[F_R \frac{\partial R}{\partial t} + F_Z \frac{\partial Z}{\partial t} + F_\Lambda \frac{\partial \Lambda}{\partial t} \right] \\ & - \int\int_{r-edge} d\theta\, d\phi \left[R \left(P_\perp + \frac{B^2}{2\mu_0} \right) \left(\frac{\partial R}{\partial \theta}\frac{\partial Z}{\partial t} - \frac{\partial Z}{\partial u}\frac{\partial R}{\partial t} \right) \right]\end{aligned}$$

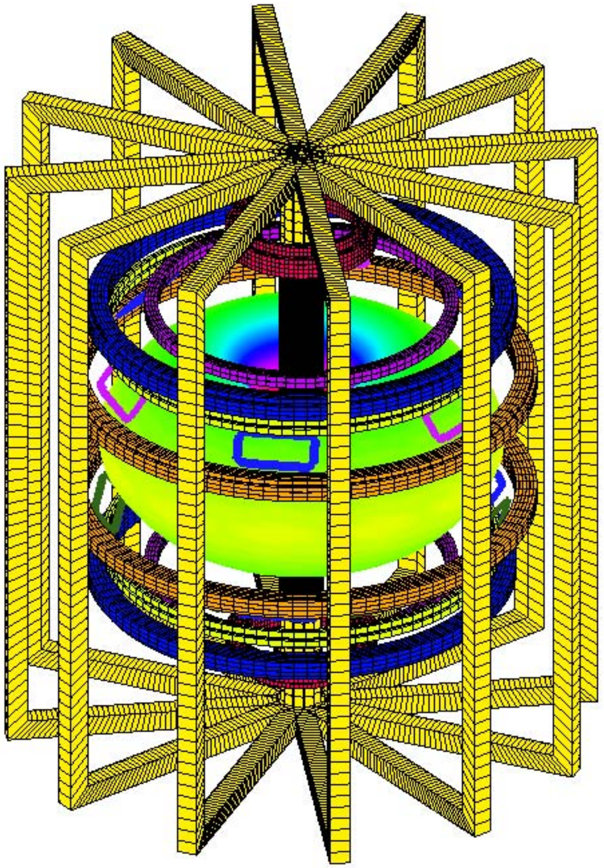
- Use Fourier decomposition in the periodic angular variables

- An accelerated steepest decent method is applied with matrix preconditioning to obtain equilibrium state.

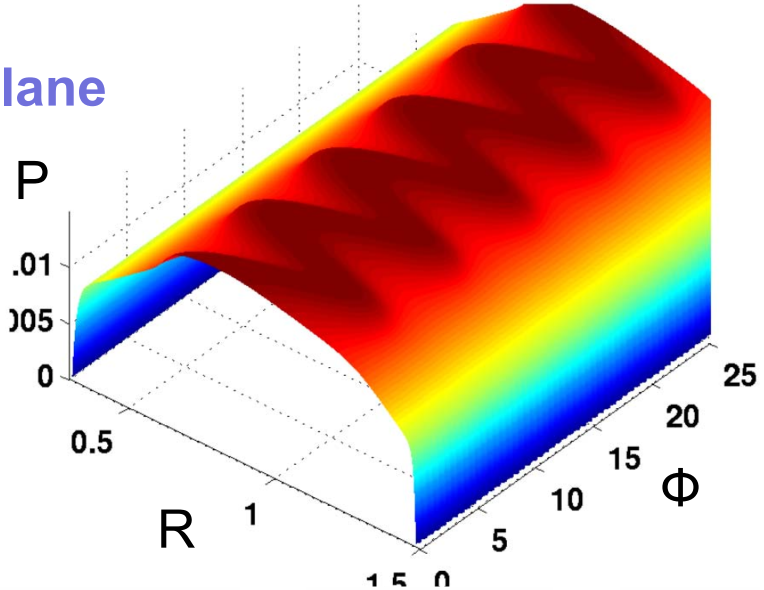
- Implemented in the ANIMEC code (anisotropic pressure extension of VMEC2000)

MAST Free Boundary (RMP +ripple)

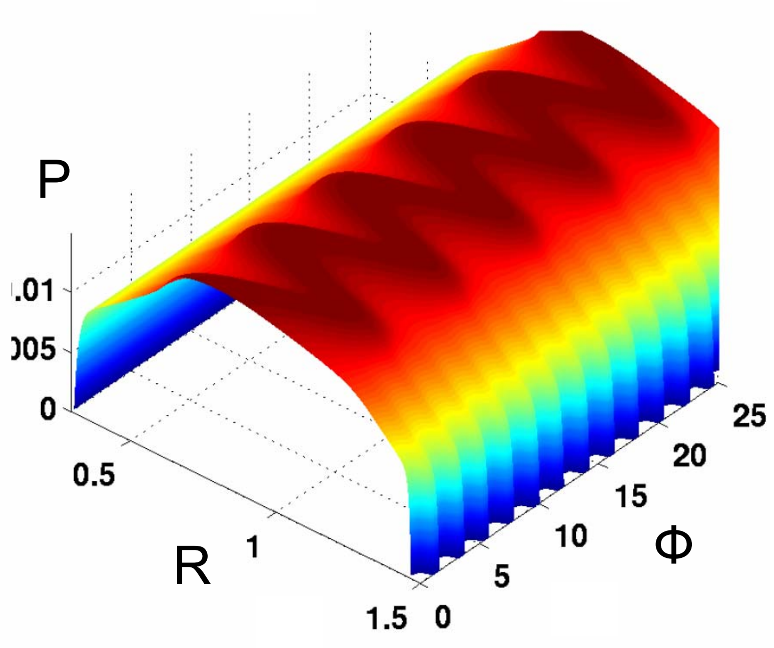
Pressure on mid-plane



Helical core +
ripple

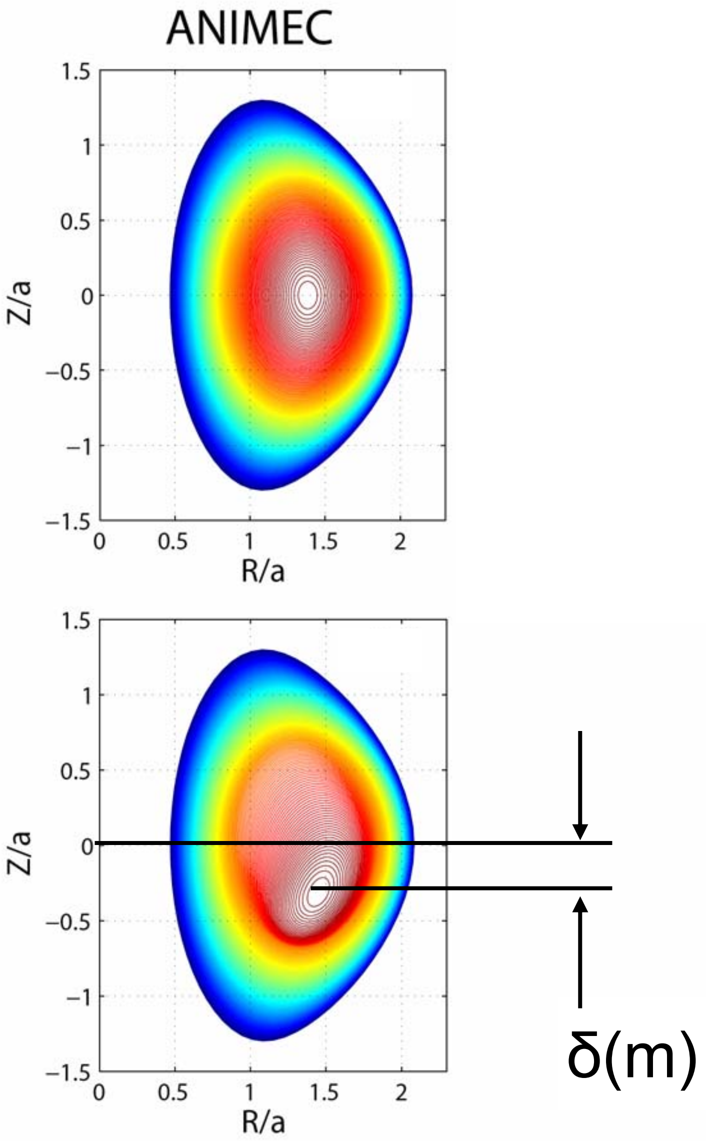


Helical core +
Ripple +
RMP (n=3)



Initial value non-linear stability code

ANIMEC
EQUILIBRIUM
CODE



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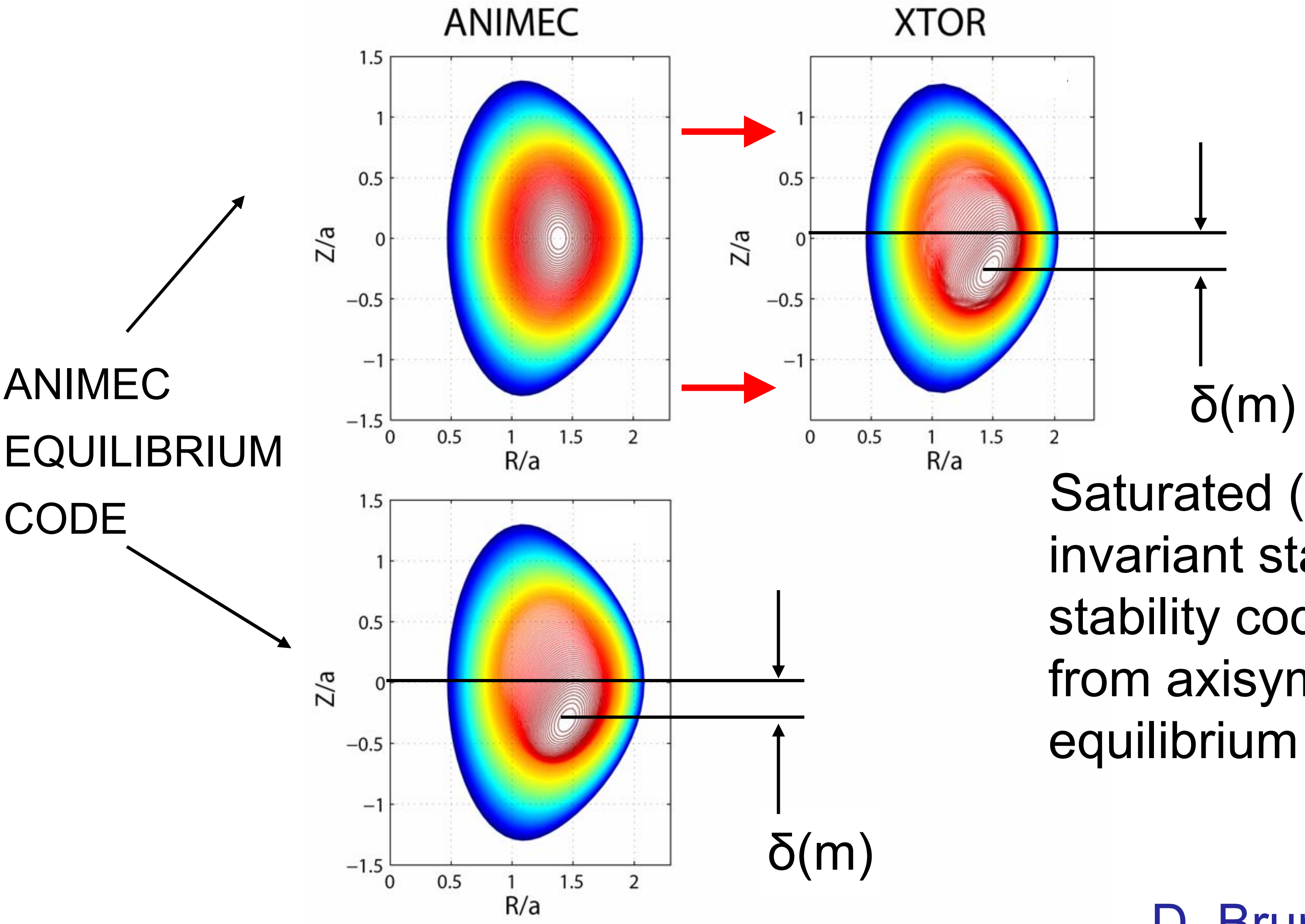
Initial value non-linear stability code



ANIMEC
EQUILIBRIUM
CODE

Saturated (time invariant state) of stability code, initiated from axisymmetric equilibrium bifurcation

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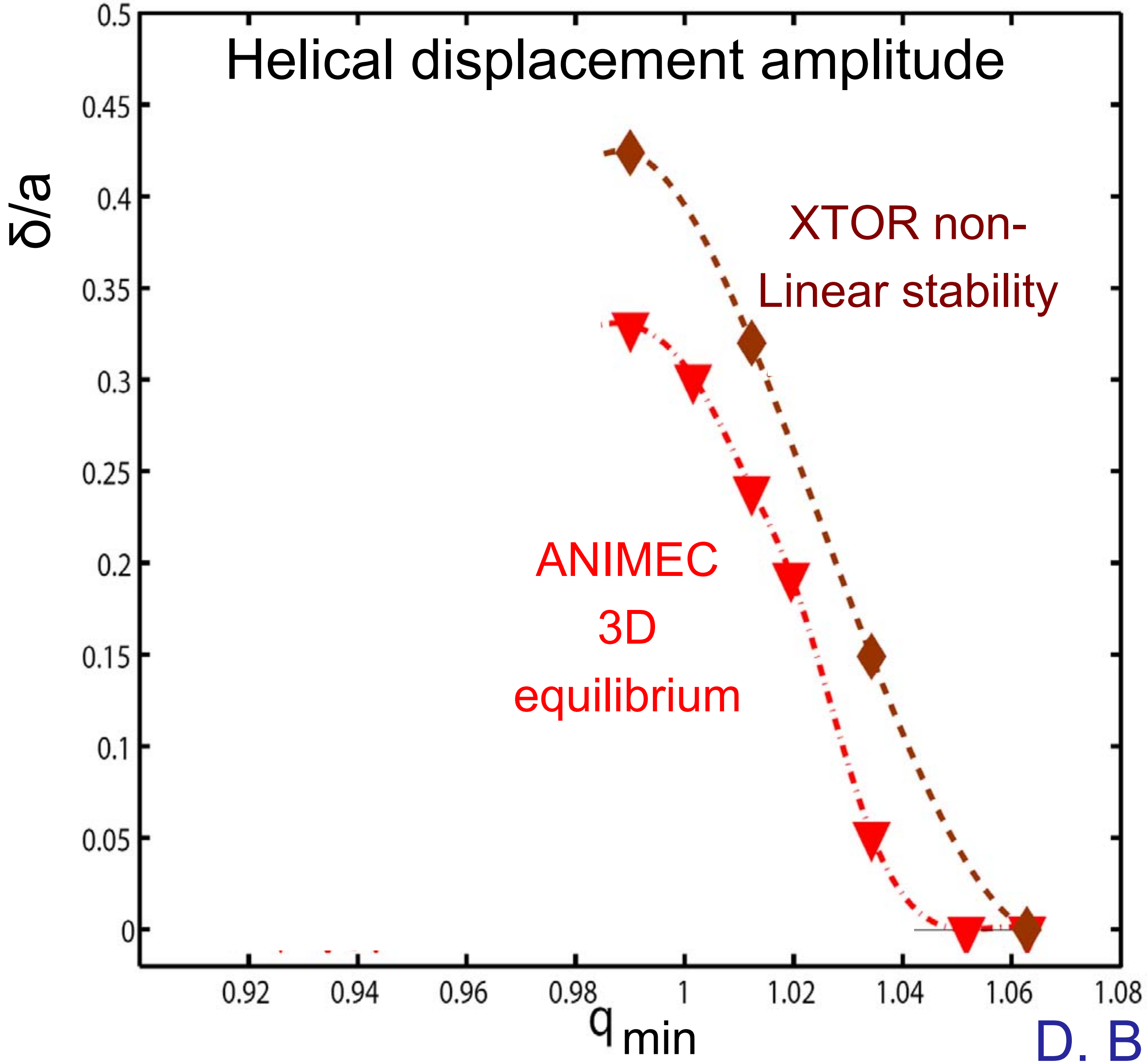
Initial value non-linear stability code



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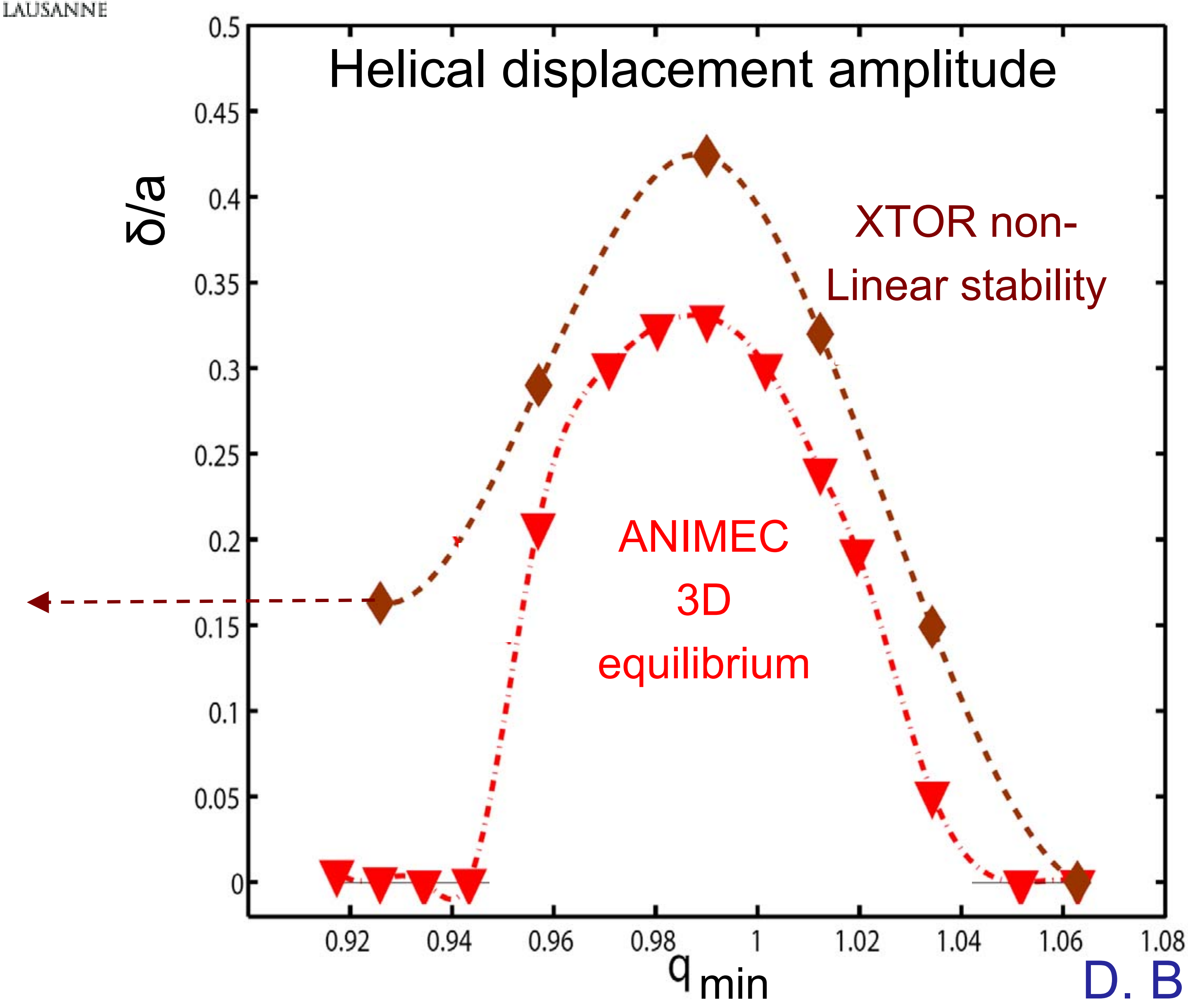
Initial value non-linear stability code



Problem for qmin<1.

Expect internal kink instability.

But no ANIMEC helical core.

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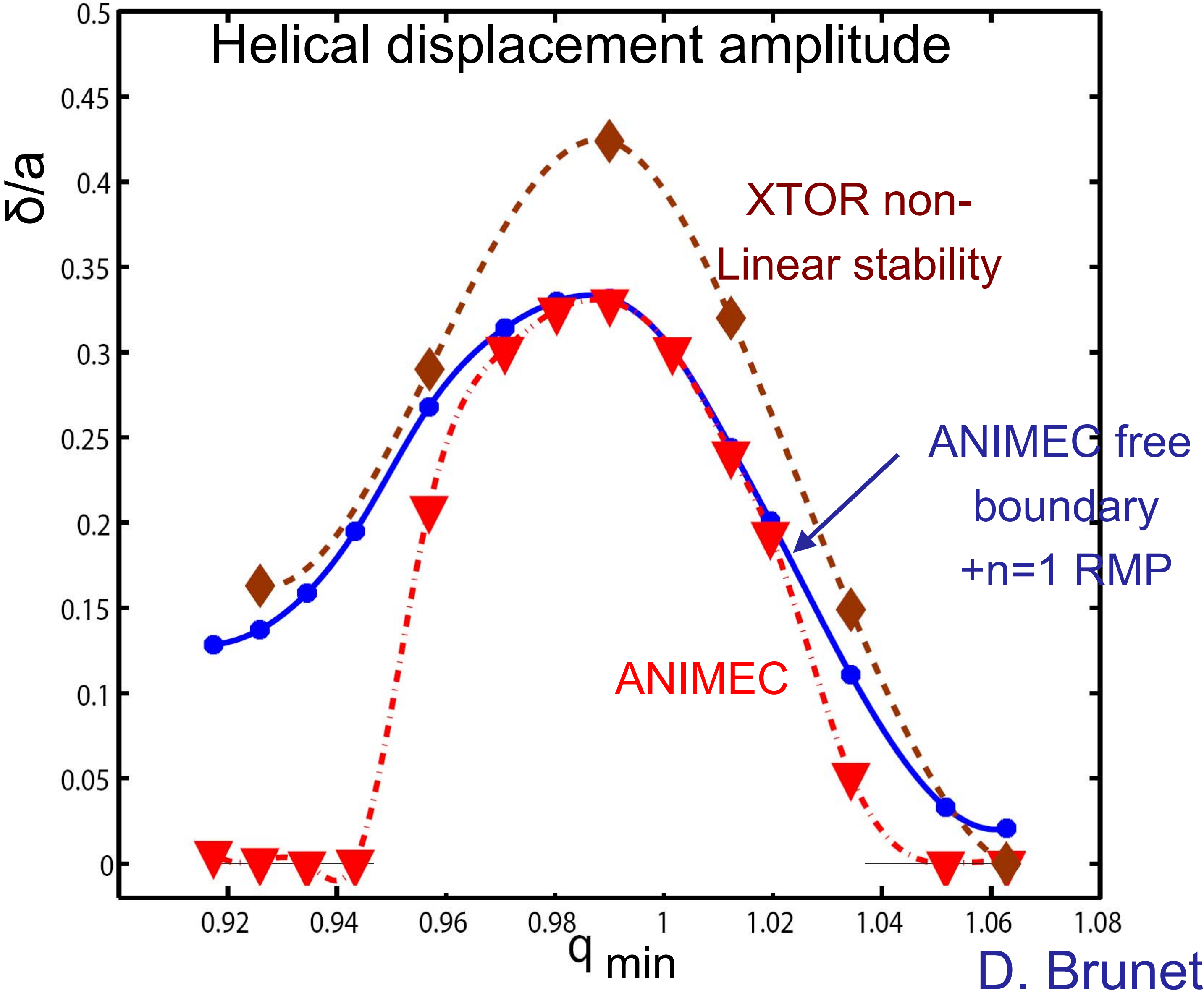
Free boundary ANIMEC + RMP



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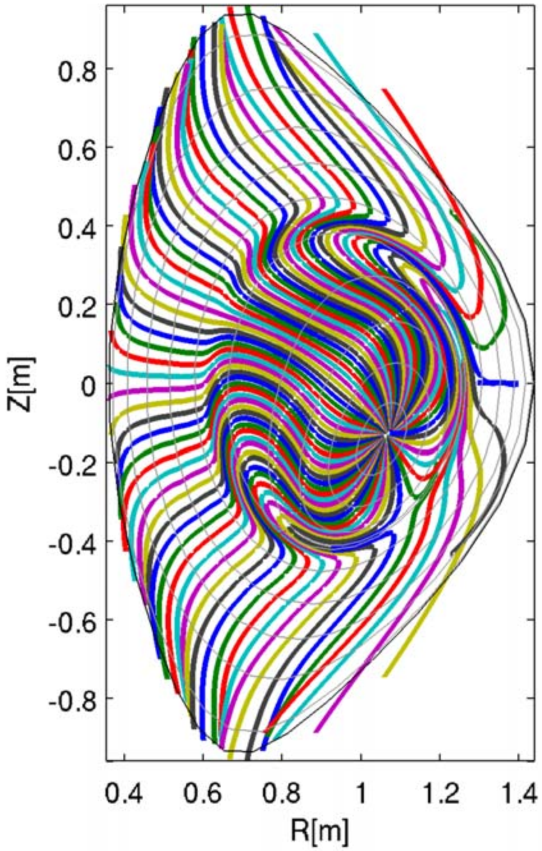
Free boundary ANIMEC + RMP

- We may use a small n=1 static field (e.g. RMP) to assist production of helical core (alternative to helical guess for magnetic axis)
- Similar approach to that used for generation of SHAX equilibria in RFP’s
- But, problem is that size of helical core depends on RMP amplitude. So this approach cannot easily be used for predictive studies for q_min>1. Work is ongoing to attempt minimisation of energy with respect to edge perturbation (analogous to stability problem).
- Nevertheless, that an equilibrium code can be used as a tool to represent saturated internal kink is very useful.
- It provides a very clean magnetic field structure for advanced modelling studies: e.g. fast particle confinement.

Coordinate Systems with Helical Core

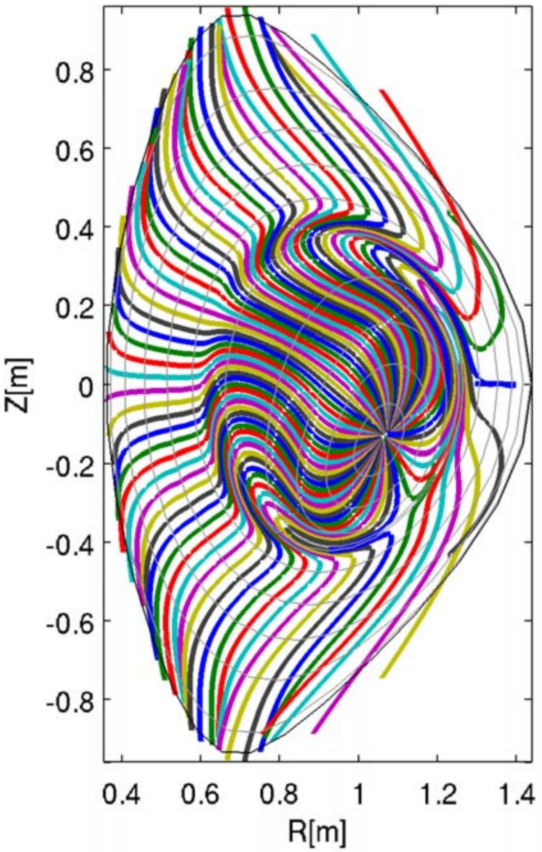
Boozer: highly non-orthogonal.
Requires up to 1000 Fourier
harmonics to represent equilibrium.

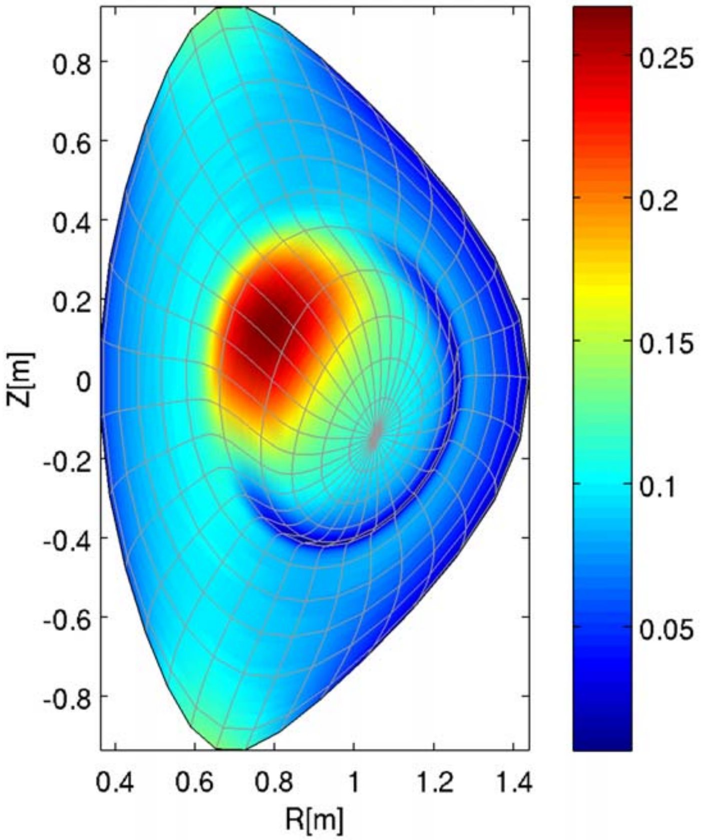


Coordinate Systems with Helical Core

Boozer: highly non-orthogonal. Requires up to 1000 Fourier harmonics to represent equilibrium.

ANIMEC: grid is more regular (about 40 Fourier harmonics), but Jacobian strongly varying.





Full F simulation of e.g NBI (or alpha) population is a tough numerical challenge

Coordinate Systems with Helical Core

Challenge: new guiding centre code *VENUS-LEVIS* employing coordinate system of equilibrium code ANIMEC. Conservation properties that satisfy both Louivilles theorem (analogous to canonical properties of Boozer based guiding centre coordinates) and full conservation of particle energy.

$$\dot{\rho}_{||} = -\partial\mathcal{H}\cdot\mathbf{B}^*/\mathbf{H}\cdot\mathbf{B}^*$$

$$\dot{\mathbf{X}} = \left(v_{||}H\mathbf{B}^* + \mathbf{H}\wedge\partial\mathcal{H}\right)/\mathbf{H}\cdot\mathbf{B}^*$$

$$\vec{\partial}\mathcal{H} = (\mu/q + v_{||}\rho_{||})\vec{\nabla}B$$

$$\vec{B}^* = \vec{B} + \rho_{||}\vec{\nabla}\wedge\vec{B}$$

Additional advantages [D. Pfefferlé, J. Phys: conf. **401**. 012020 (2012)] :-

•Exploits inherent Fourier representation in poloidal and toroidal angles, and in corresponding derivatives (non-discretised).

•Retains effect of full magnetic field, including the radial equilibrium covariant field. Thus configurations leading to strongly non-orthogonal coordinates can be handled (unlike codes based on Boozer coordinates).

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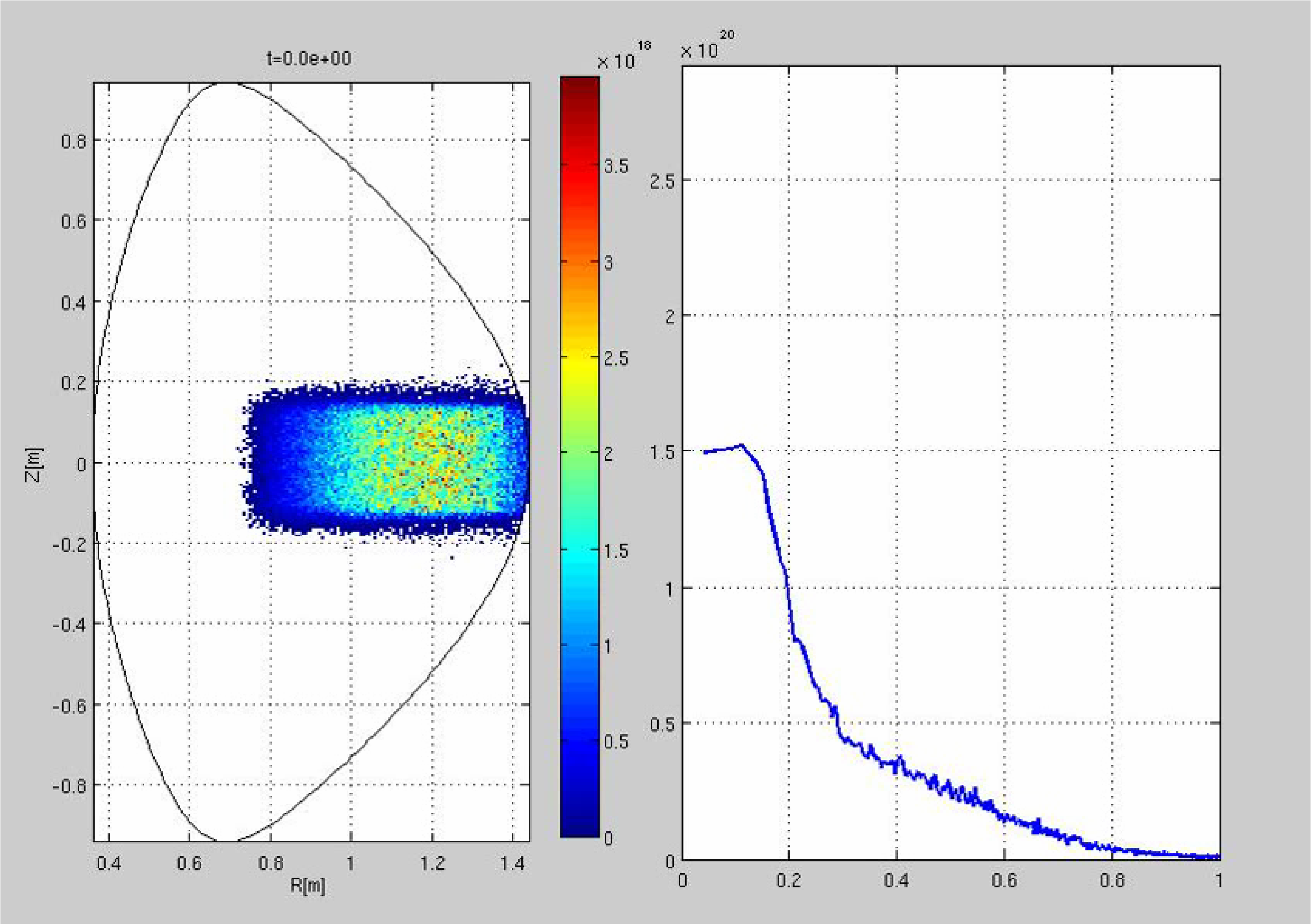
MAST NBI modelling

Full F early phase: axisymmetric case



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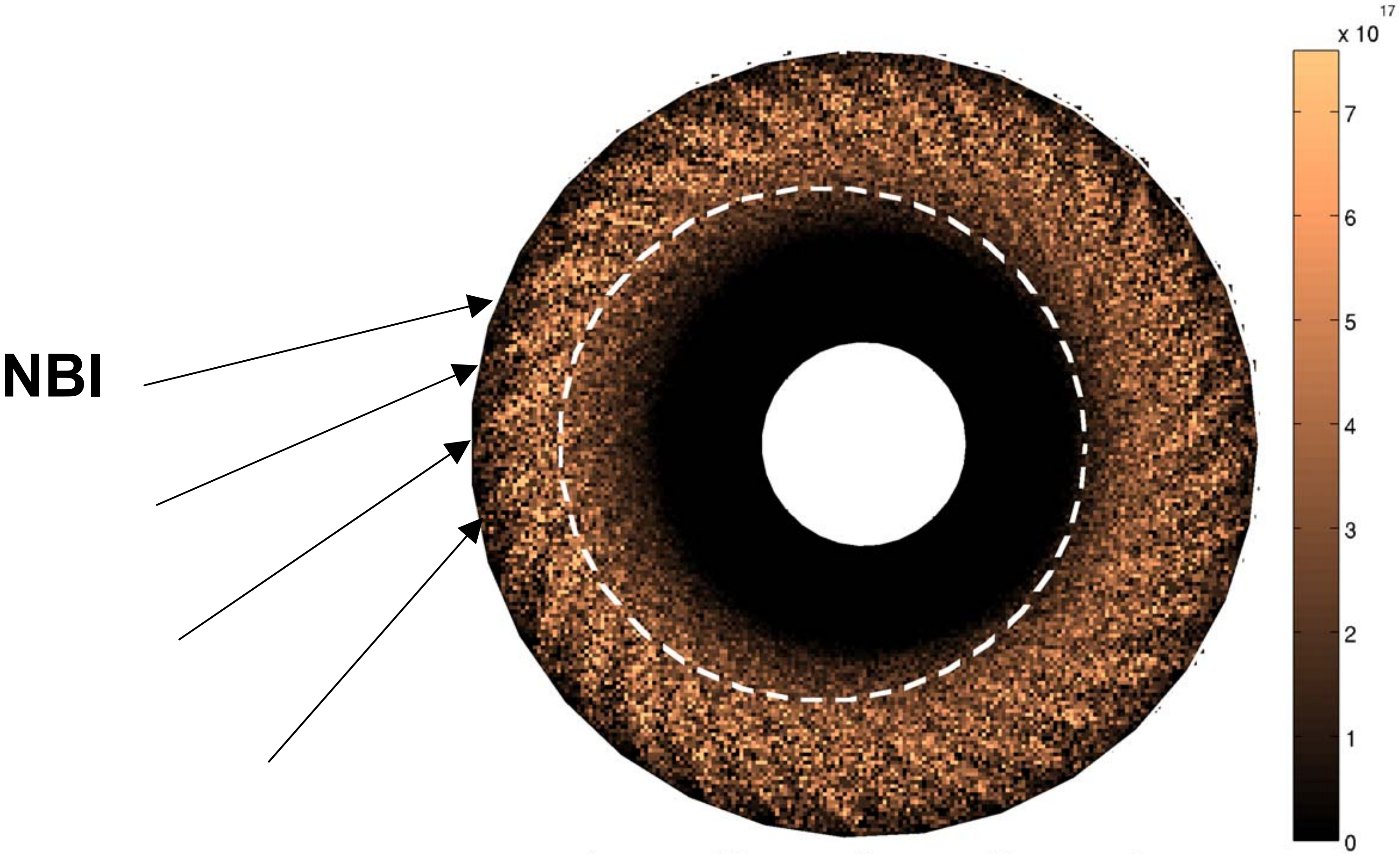
MAST NBI modelling

Initial injection: helical (LLM) case, and multiple injectors to model rotating helical core



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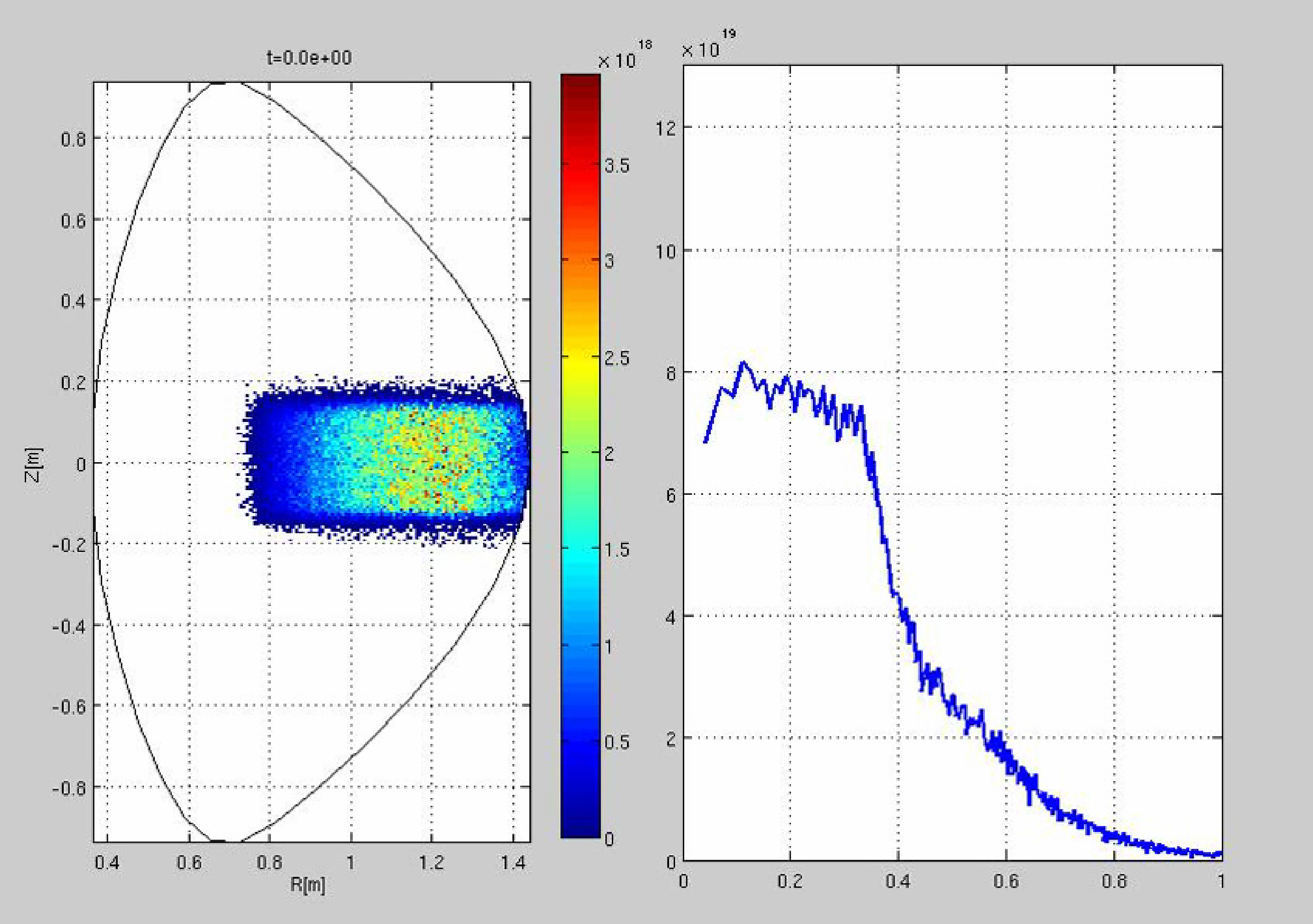
MAST NBI modelling

Full F early phase: helical (LLM) case



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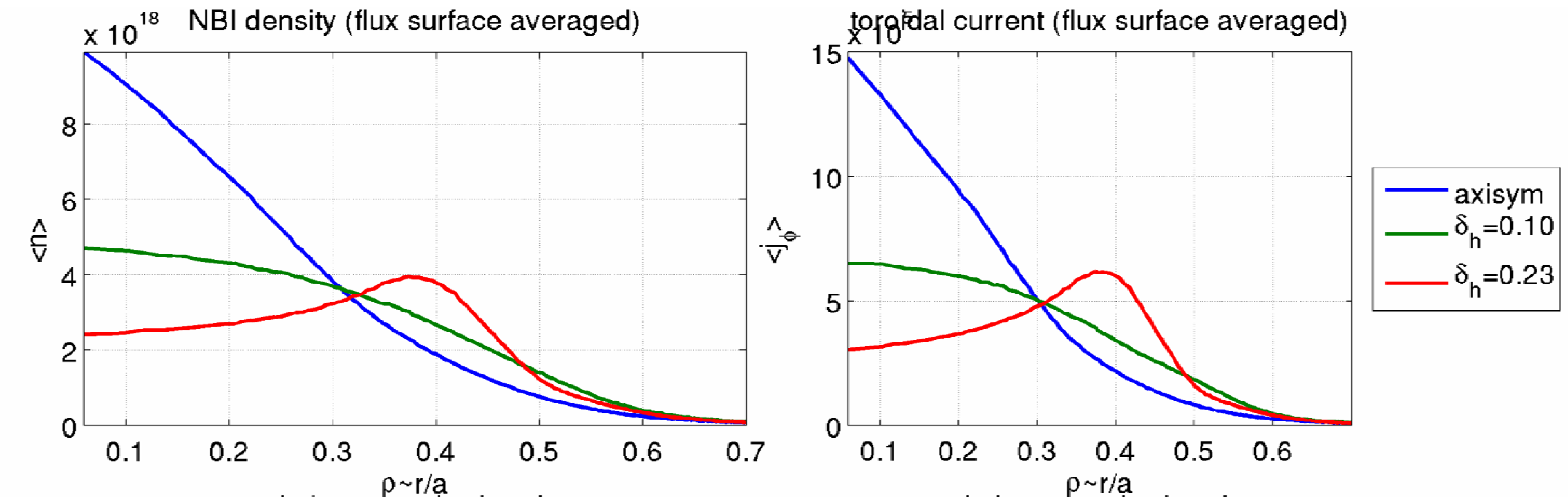
MAST NBI modelling



Particles deposited off axis because the LLM moves the axis relative to the NBI injection.

Total number of confined NBI ions almost the same with or without LLM. But heating and current drive off axis.

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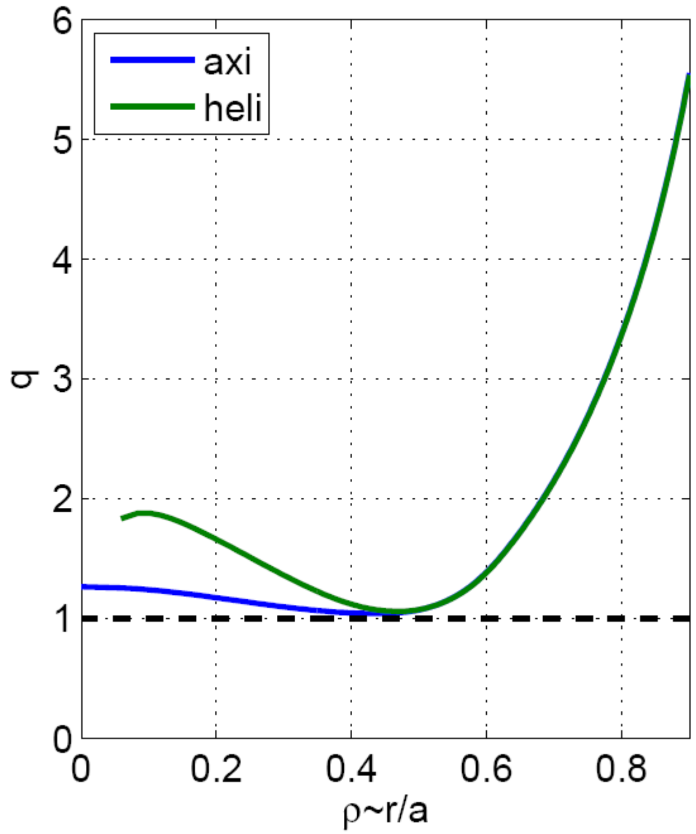
MAST NBI modelling



Current drive off axis: modelled effect on q-profile

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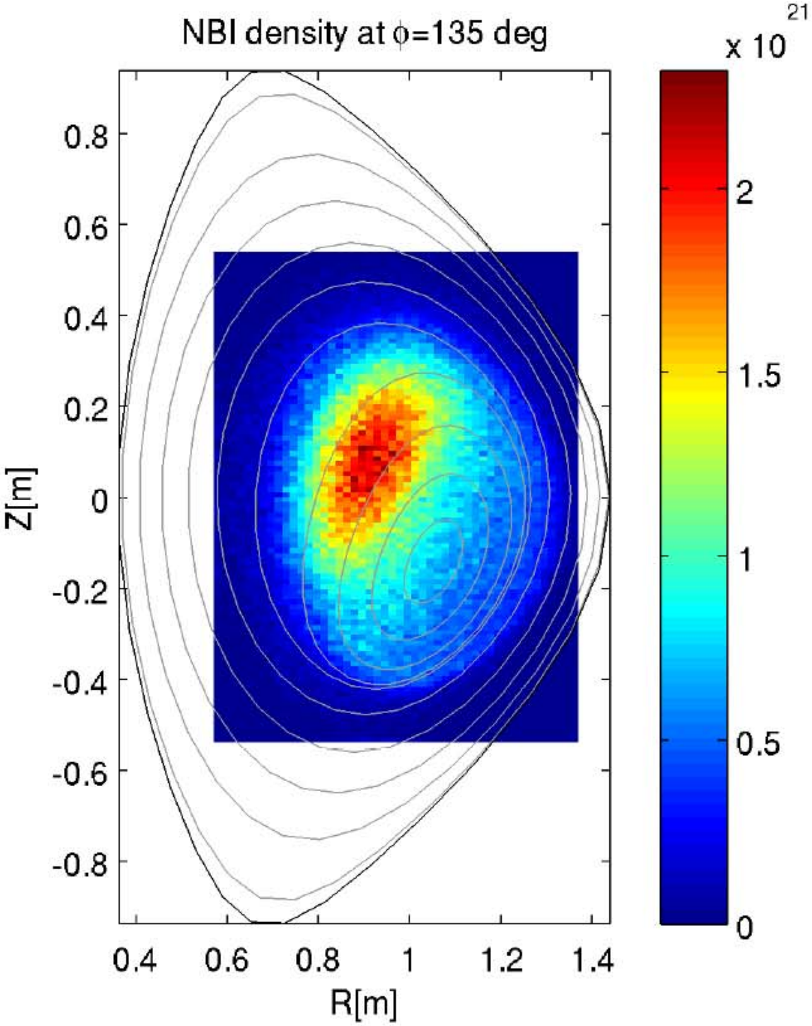
Fast ion density traces a snake



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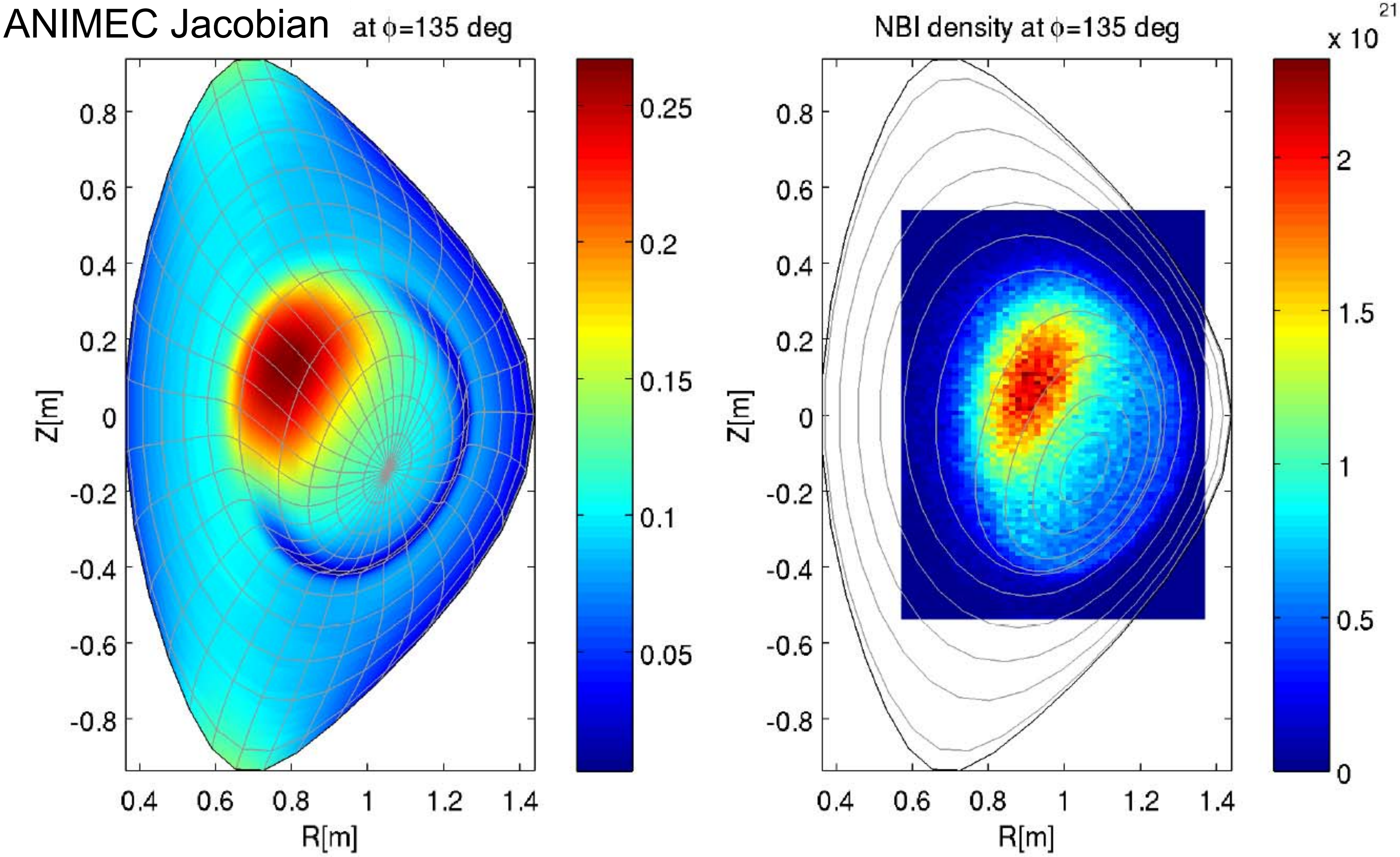
Fast ion density traces a snake

~ ANIMEC Jacobian



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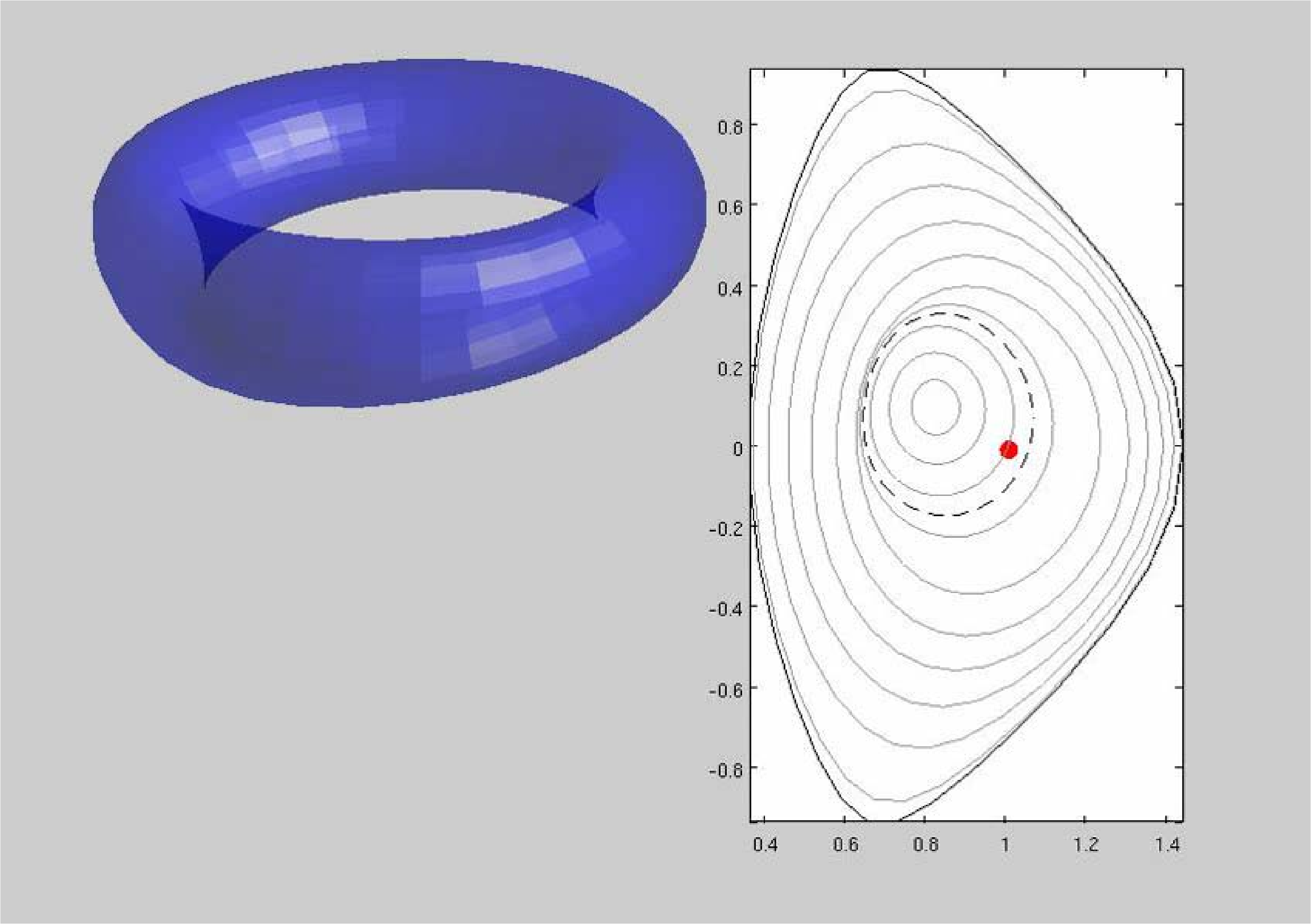
Exotic Particle Orbits generate snake



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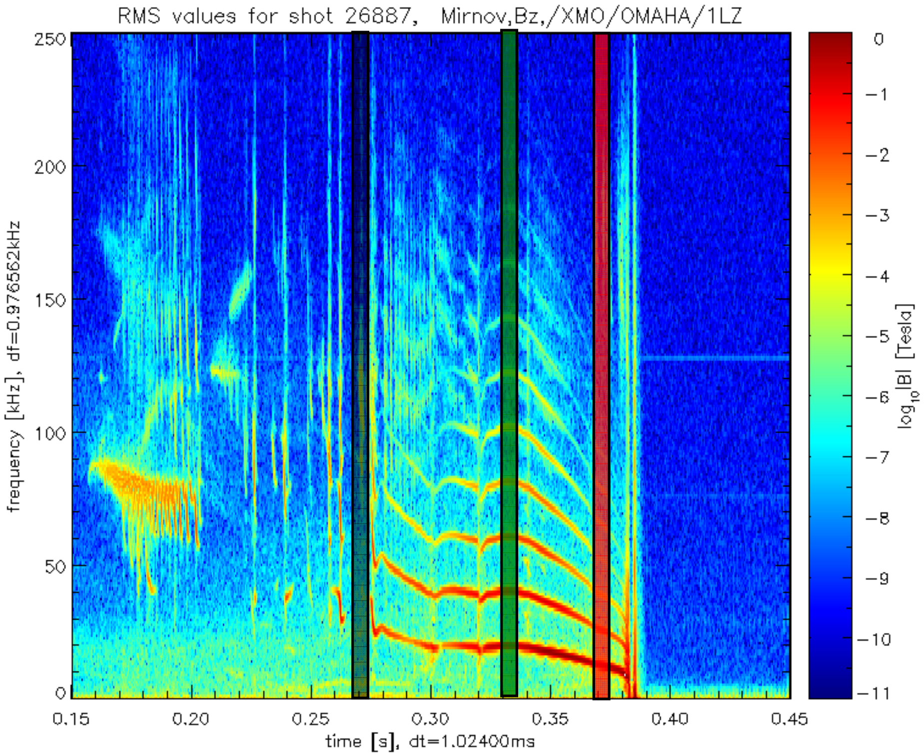
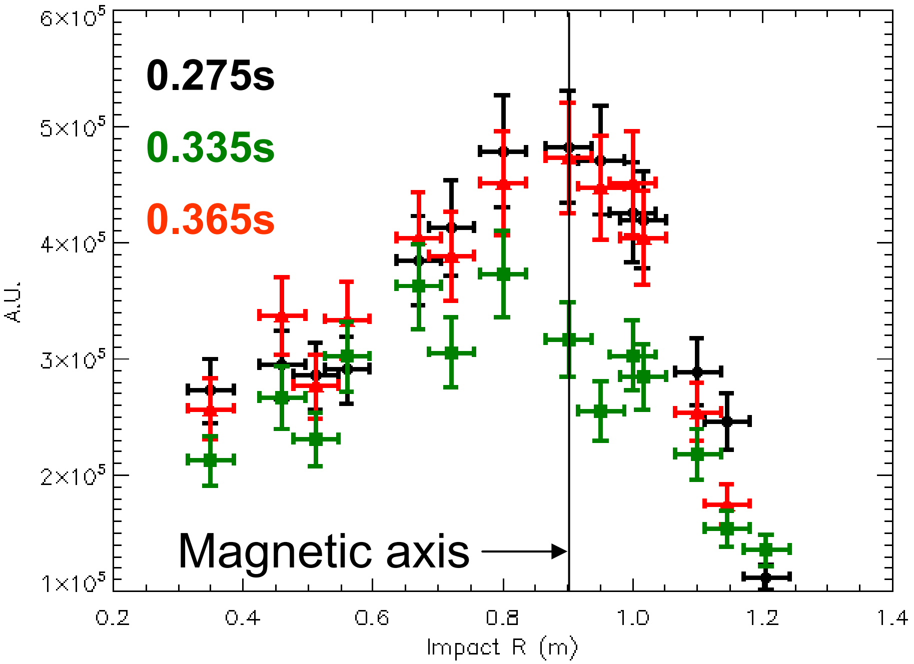
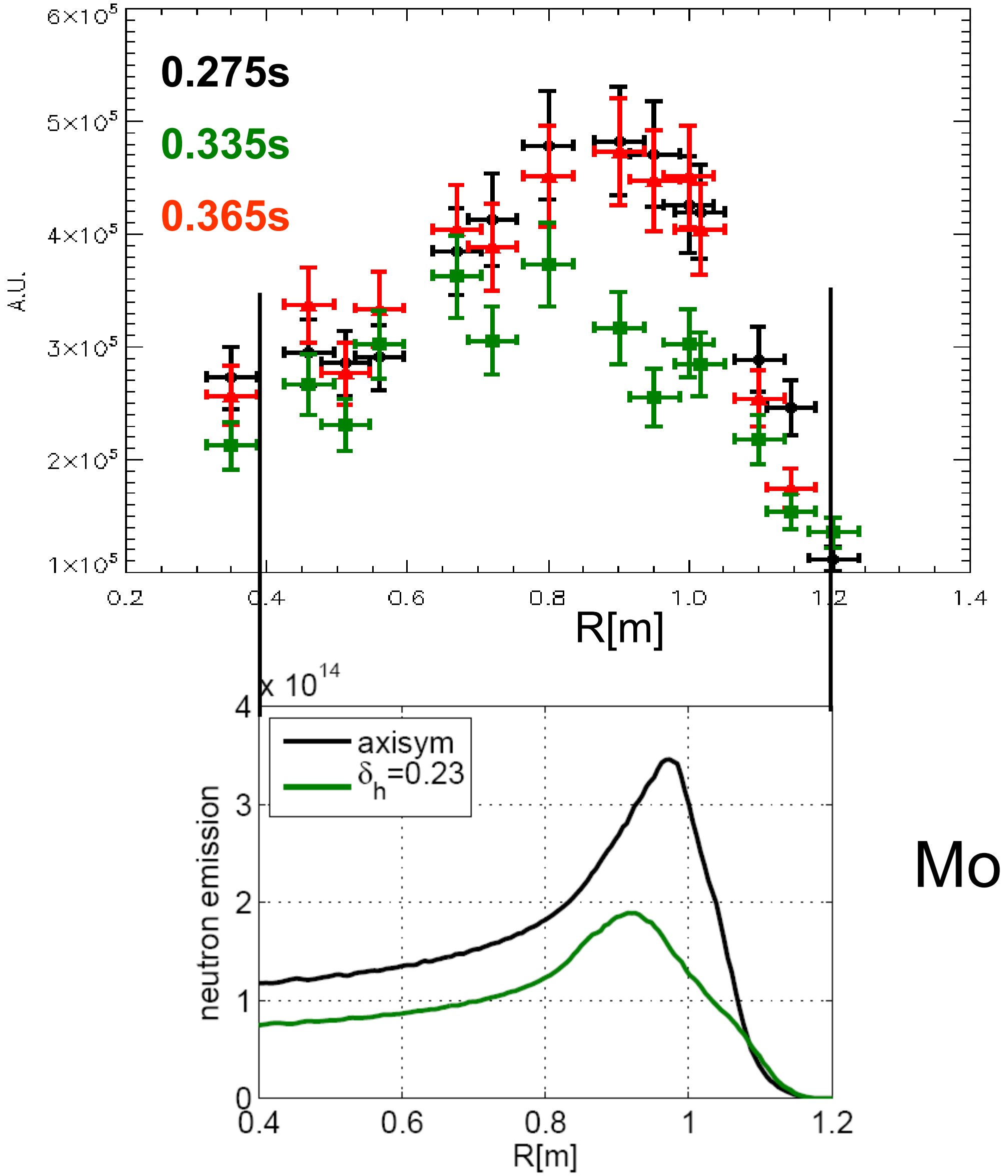
Neutron emissivity profiles in MAST

- Fast ion redistribution follows q-profile evolution
  - As q_{min} approaches unity, LLM appears and fast ions are expelled from the plasma core (fast ions distribution represented by neutron emissivity)
  - As q_{min} drops through unity, internal mode growth drops (alternatively, helical core amplitude decreases) and fast ions confined once more





Comparison with Neutron Emmisivity: virtual diagnostic



EXP DATA

Modelling (virtual diagnostic)

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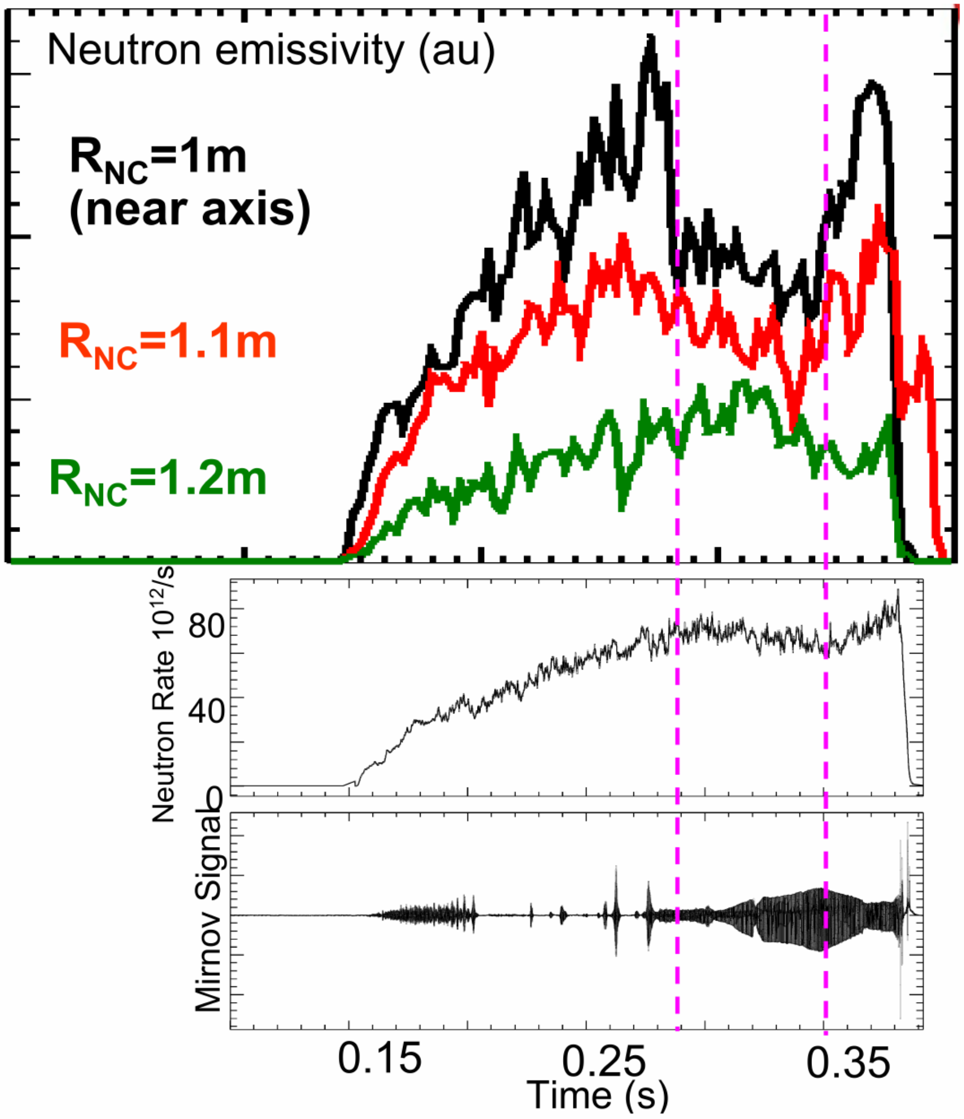
Comparison with virtual diagnostic



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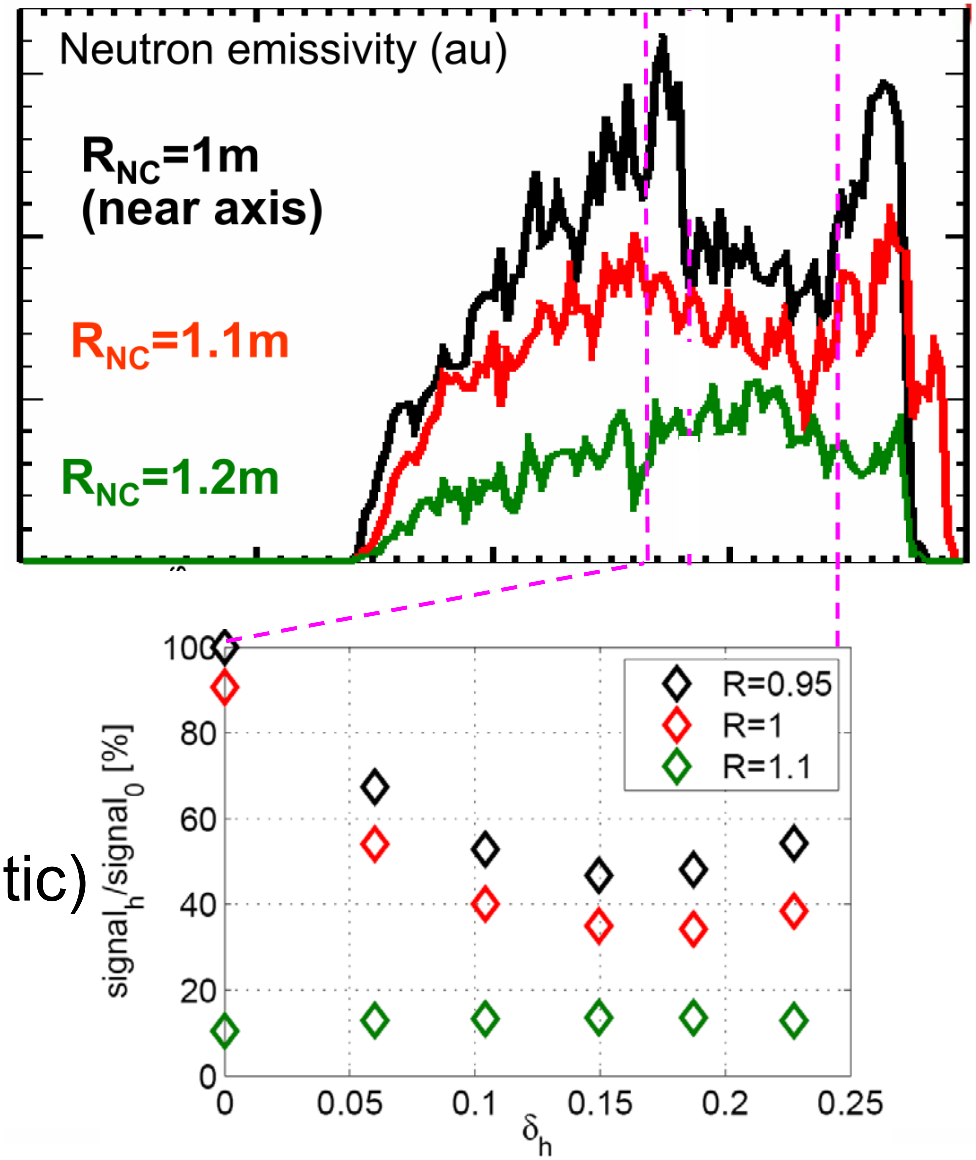
Comparison with virtual diagnostic

EXP DATA

Modelling (virtual diagnostic)



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Conclusions

- Despite the assumption of an axisymmetric boundary, it is found that an equilibrium can be non-axisymmetric in the core. Such phenomenology is consistent with saturated n=1 modes in hybrid scenarios of many tokamaks.

- Employment of 3D equilibrium code ANIMEC, usually reserved for stellarator physics, indicates two bifurcations for non-monotonic q profiles (providing $q_{\rm min} \approx 1$).

- Use of n=1 RMP with free boundary equilibria enables equilibrium code to represent helical saturated modes also for $q_{\rm min} < 1$.

- Fast ion orbit confinement properties established with guiding centre code capable of handling extreme but cleanly represented geometry.

- In presence of helical core, nominal on-axis NBI deposition becomes off-axis. Heating and current drive consequently modified.

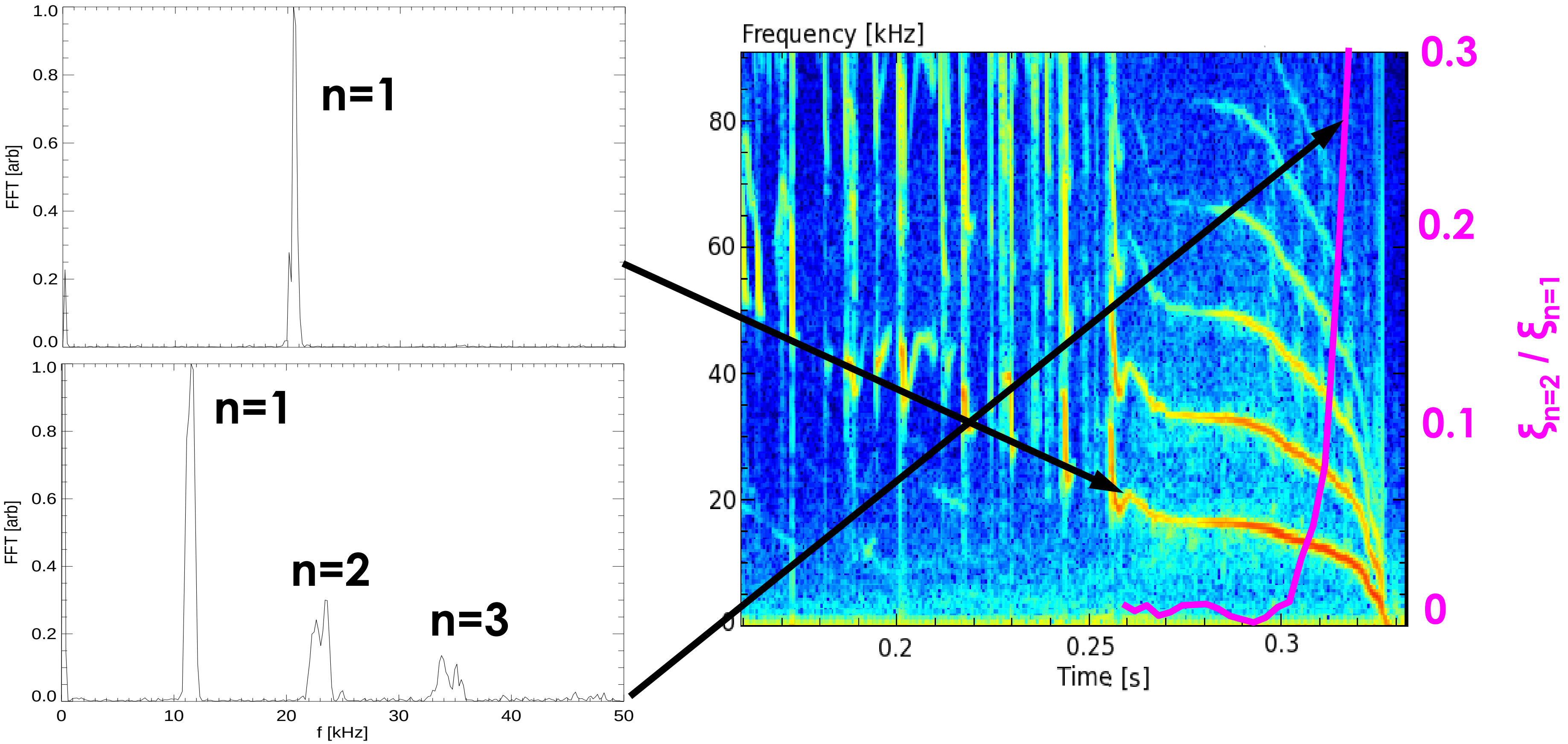
- Fast ion orbits are exotic, leading to strong local variation of heat and current.

- Important consequences for hybrid scenarios of ITER.

Additional Material

Higher-n harmonics of LLM

- LLM is observed to be n=1 at onset
    - Relative amplitude of n=1 and n=2 harmonics from SXR changes
    - Perhaps RFA of n=2 harmonic arising nonlinearly in presence of LLM when the n=2 infernal mode becomes marginally unstable



Hybrid Scenario in JET

- Continuous core localised ideal n=1 kink modes are observed in JET hybrid configuration [Buratti et al, NF 2012].
- These ideal modes significantly reduce confinement, and usually precedes an NTM.