Plasma rotation: momentum transport and 3D effects

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Thanks to X. Garbet and A.G. Peeters

Outline

- **Part I**
  - Axisymmetric fields (2D): turbulent transport
- **Part II**
  - 3D fields: NTV
- **Part III**
  - Which effect dominates?

Disclaimer: I am not a NTV expert...part of this talk may well be quite naive!!

Before starting...

- To discuss rotation, a **momentum evolution equation** is needed
- Ideally, this evolution equation incorporates all possible/important mechanisms → useful framework derived by Callen, APS'09
- Macroscopic quantities from moments of the distribution function:

density $$n = \int f \,\mathrm{d}\mathbf{v}$$

flow $$\mathbf{u} = \frac{1}{n}\int \mathbf{v} f \,\mathrm{d}\mathbf{v}$$

pressure tensor $$\underline{\underline{\pi}} = \int m(\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u}) f \,\mathrm{d}\mathbf{v}$$

- Evolution given by the Fokker-Planck equation
- Split the distribution function according to various ordering:

$$f = \bar{f}_0 + \underbrace{\bar{f}_1 + \ldots}_{\text{NTV}} + \underbrace{\tilde{f}_1 + \ldots}_{\text{turbulence}}$$

Toroidal angular momentum evolution

- Assume small non-axisymmetry and flux surfaces exist
- Take the momentum equation and:
  - sum over species with $m_e \ll m_i$
  - toroidal projection
  - flux surface average $< . >$
  - incompressible flows
  - consider transport time scales (slow)
  - focus on NTV (non-resonant) and turbulent transport:
    (neglect resonant JxB, cross-field neo. transport and sources)

$$\frac{\partial}{\partial t} < mnR\mathbf{e}_\varphi \cdot \mathbf{u} > \sim \; - < R\mathbf{e}_\varphi \cdot \nabla \cdot \underline{\underline{\pi}}_{i\|} > - \frac{1}{V'} \frac{\partial}{\partial r} \left[V' < \Pi_{r\varphi}^{\mathrm{turb}} > \right]$$

NTV $\qquad\qquad$ turbulence

$$\sim < (p_\| - p_\perp) \frac{1}{B} \frac{\partial B}{\partial \varphi} > \; \sim < nmR\tilde{u}_r\tilde{u}_\varphi + mRu_\varphi\tilde{u}_r\tilde{n} >$$

[see e.g. Callen NF'09]

Part I - Turbulent transport

- Assume an axisymmetric field → no NTV
- Momentum flux carried by the particle flux neglected

$$\frac{\partial}{\partial t} < mnR\mathbf{e}_\varphi \cdot \mathbf{u} > \sim \cancel{- < R\mathbf{e}_\varphi \cdot \nabla \cdot \underline{\underline{\pi}}_{i\|} >} - \frac{1}{V'}\frac{\partial}{\partial r}\left[V' < \Pi_{r\varphi}^{\mathrm{turb}} >\right]$$

NTV

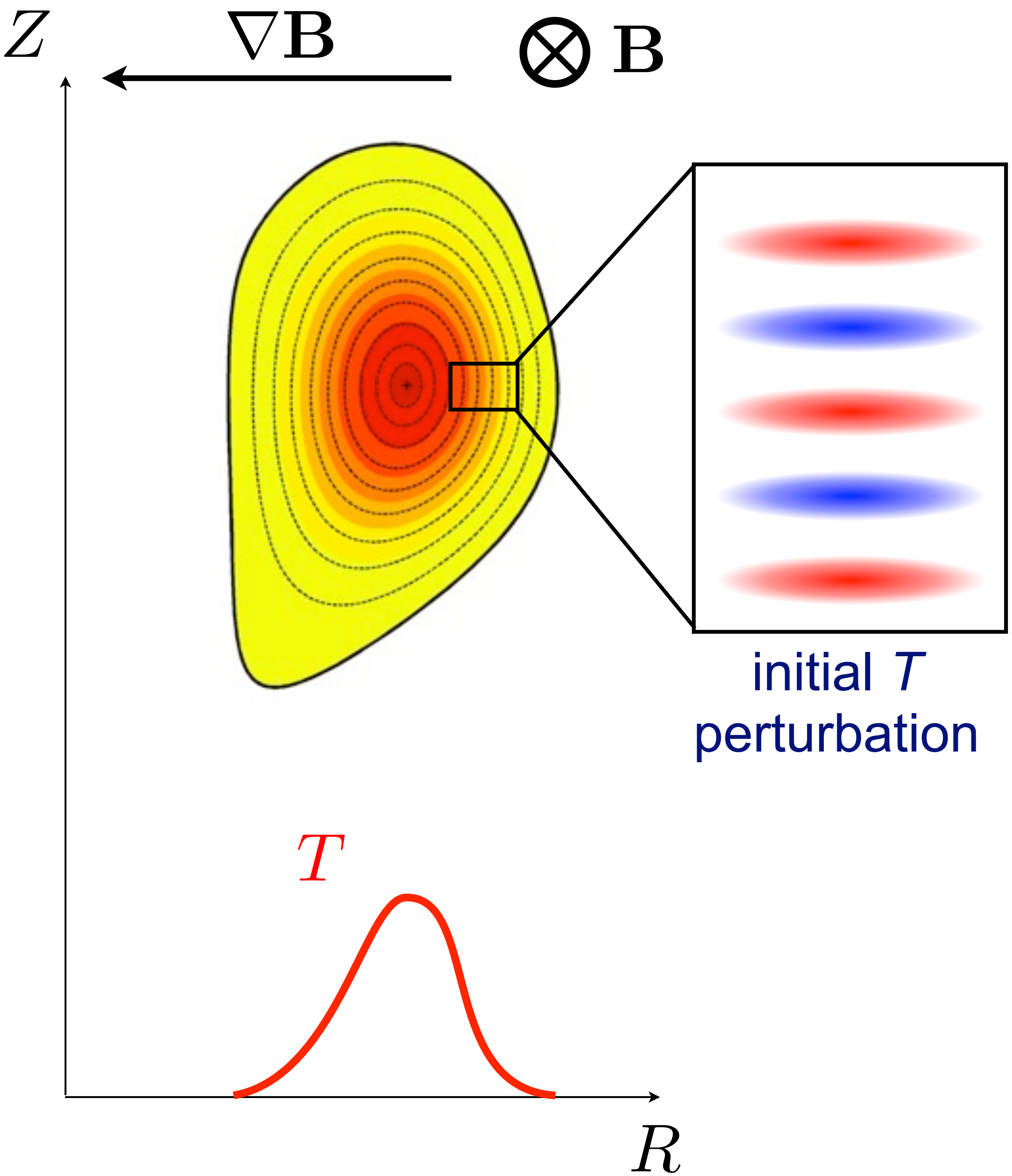
$$\cancel{\sim < (p_\| - p_\perp)\frac{1}{B}\frac{\partial B}{\partial \varphi} >}$$

turbulence

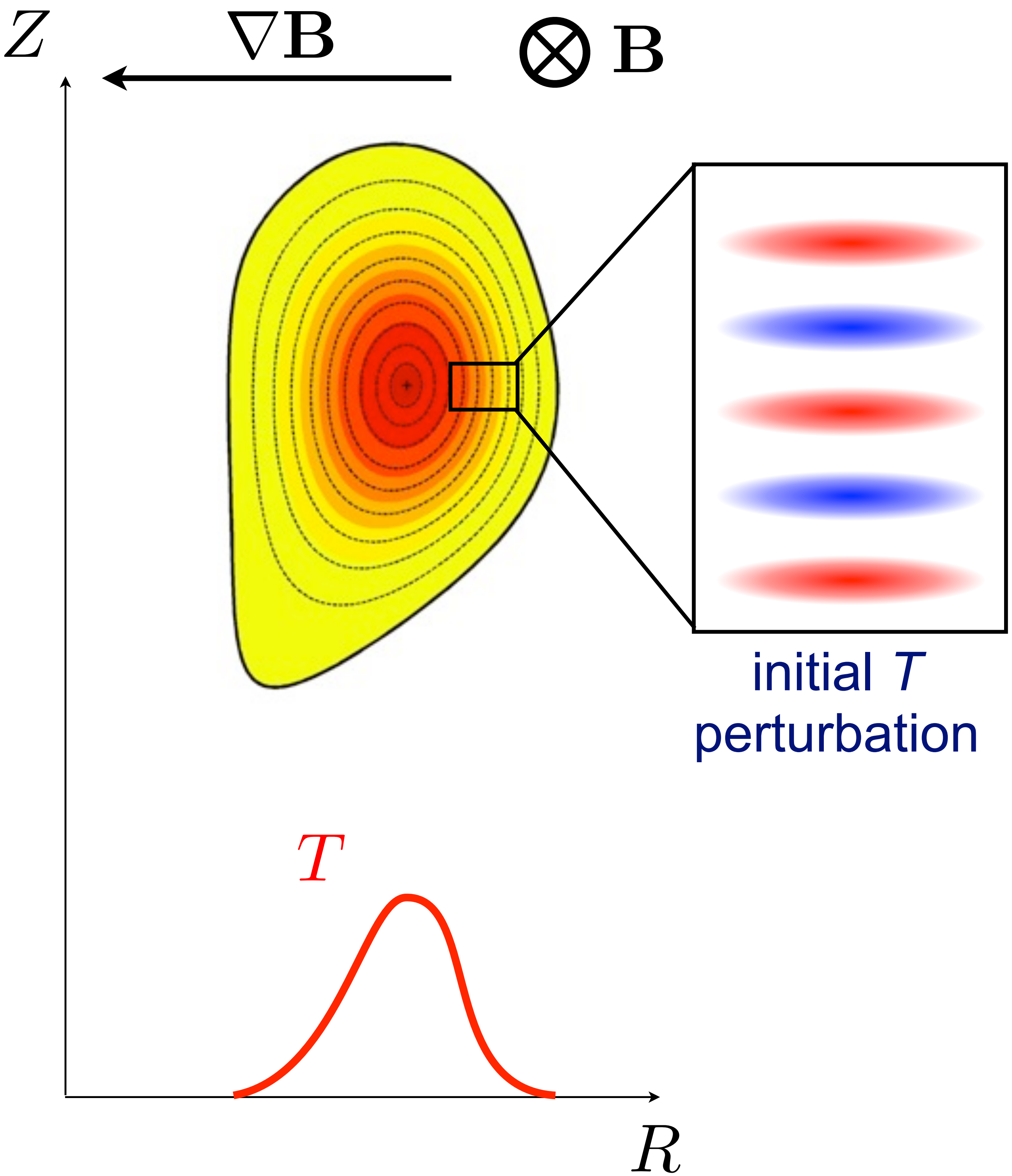
$$\sim < \boxed{nmR\tilde{u}_r\tilde{u}_\varphi \to \Gamma_\varphi} + mRu_\varphi\tilde{u}_r\tilde{n} >$$

- **In the following:**
  - Simple picture of the toroidal ITG
  - Momentum flux driven by the toroidal rotation gradient (diag. part)
  - Momentum flux driven by the toroidal rotation (pinch part)
  - A word on residual stress & summary table

Simple picture of the toroidal ITG



Magnetic drifts



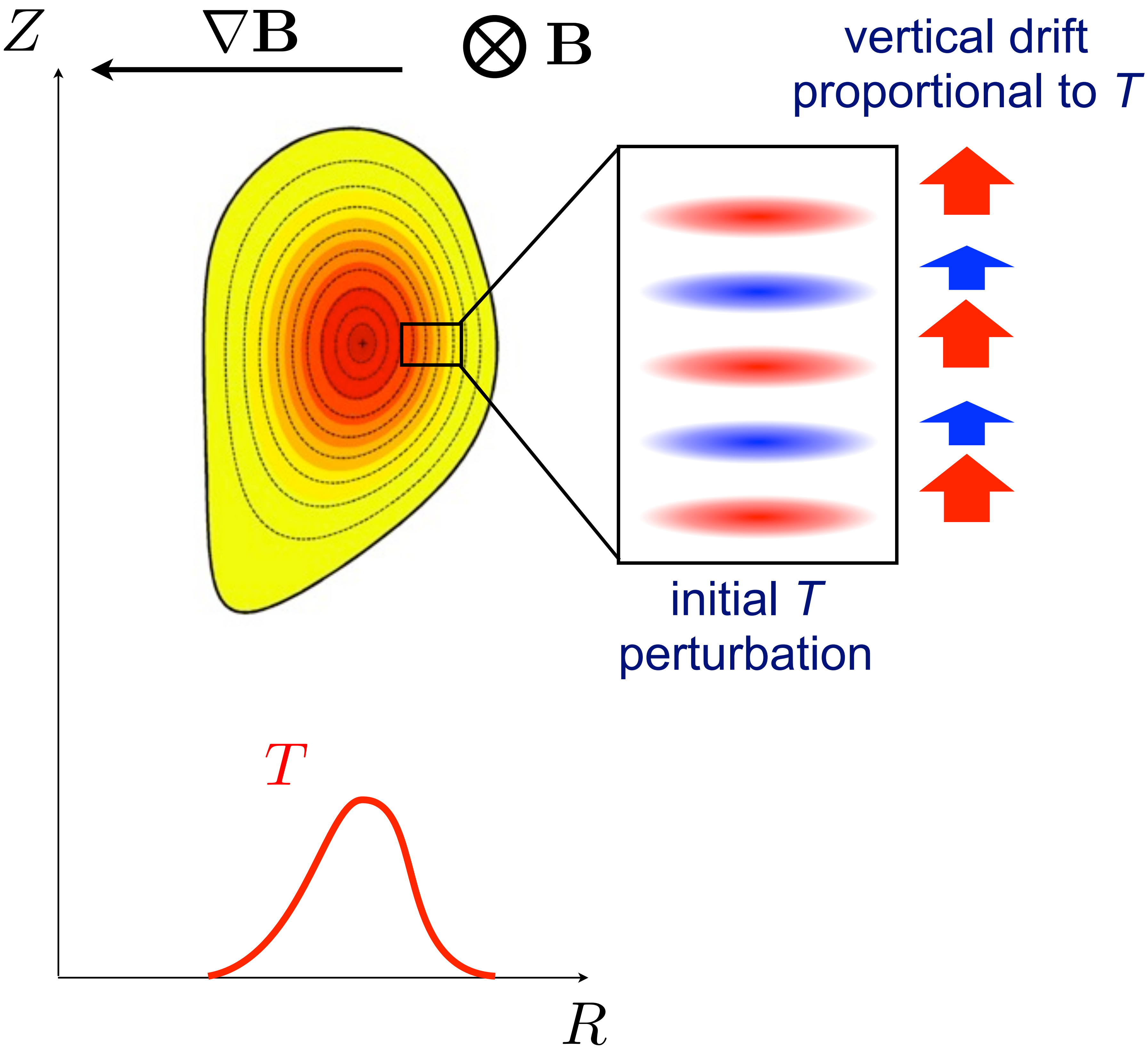
- Inhomogeneous magnetic field leads to curvature and ∇B drift

$$v_d = \frac{m}{ZeB}\left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\frac{\mathbf{B}\times\nabla B}{B^2}$$

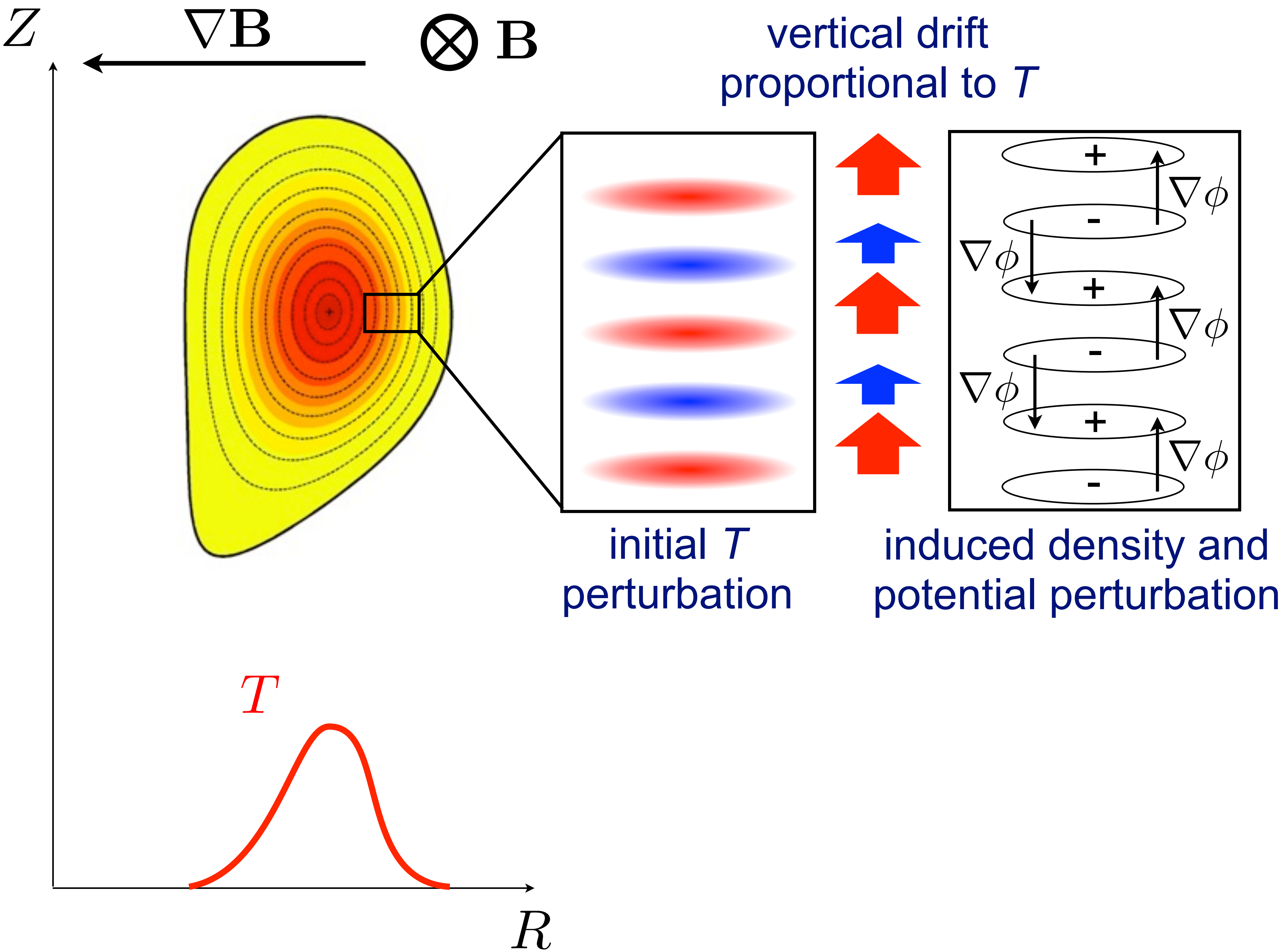
- Drift in the vertical direction and proportional to T

$$v_d^f \propto T$$

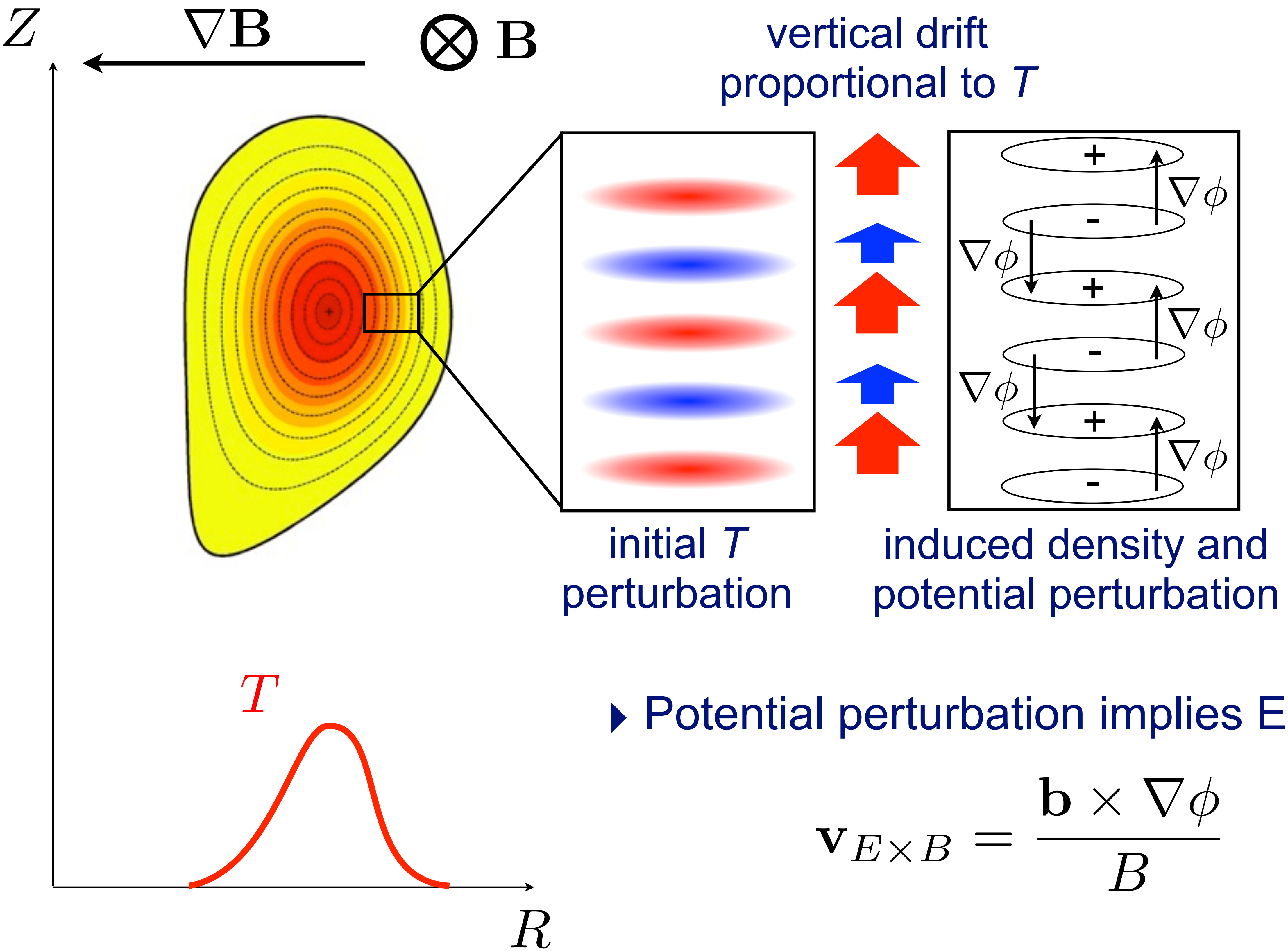
Magnetic drifts



Compression → density perturbation



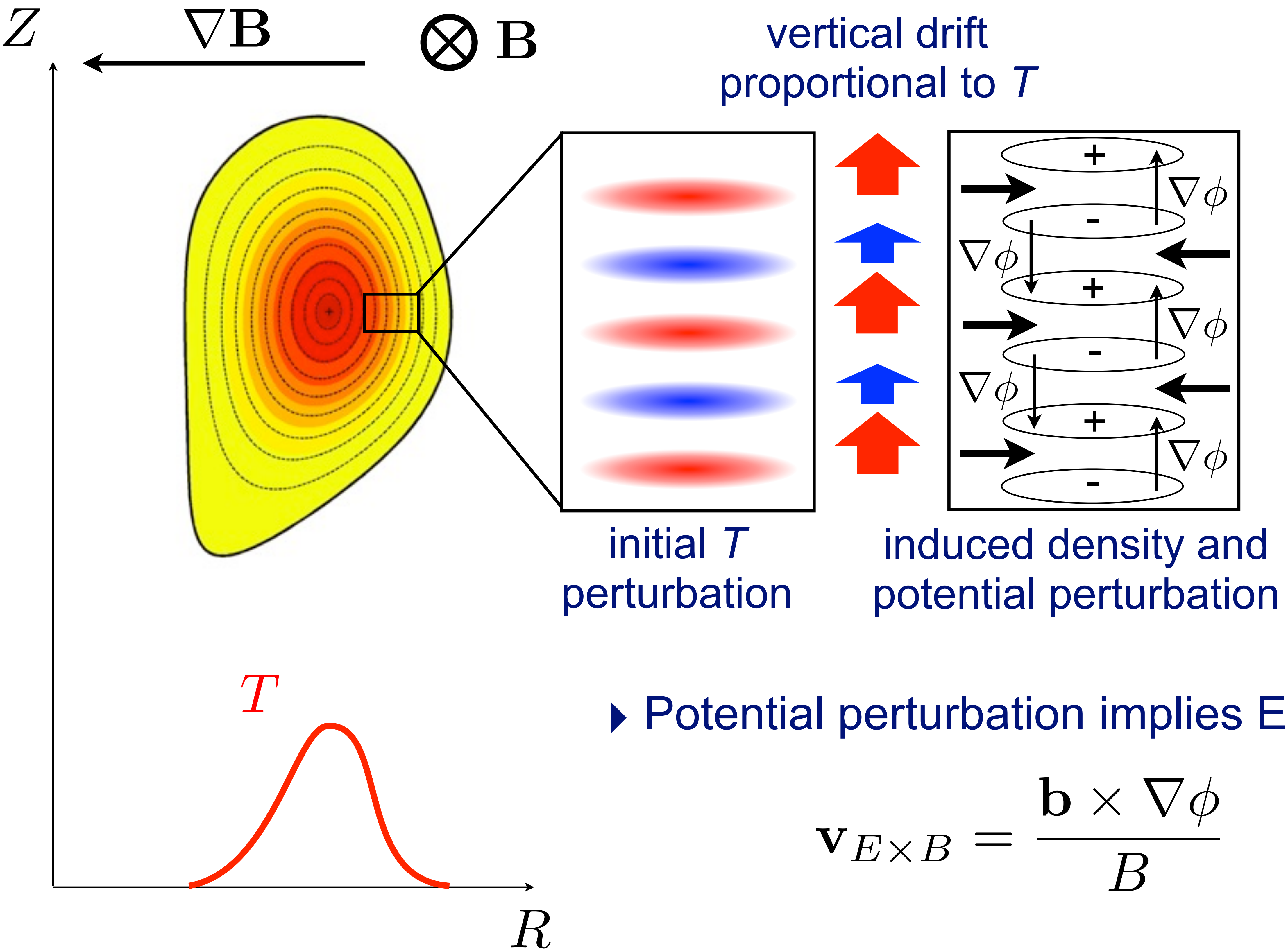
Potential perturbation → ExB drift



- Potential perturbation implies ExB drift:

$$\mathbf{v}_{E\times B} = \frac{\mathbf{b}\times\nabla\phi}{B}$$

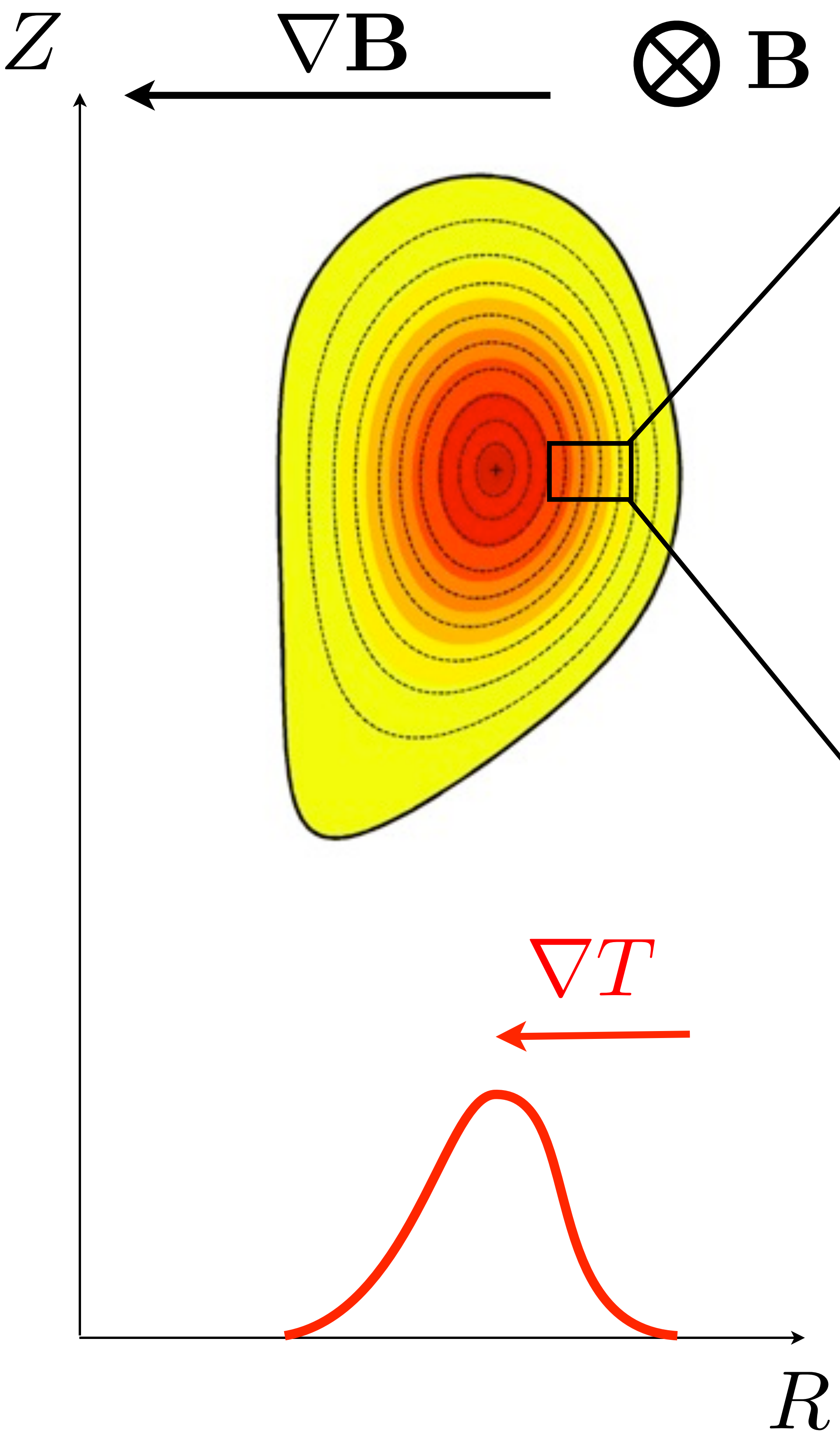
ExB drift → radial convection

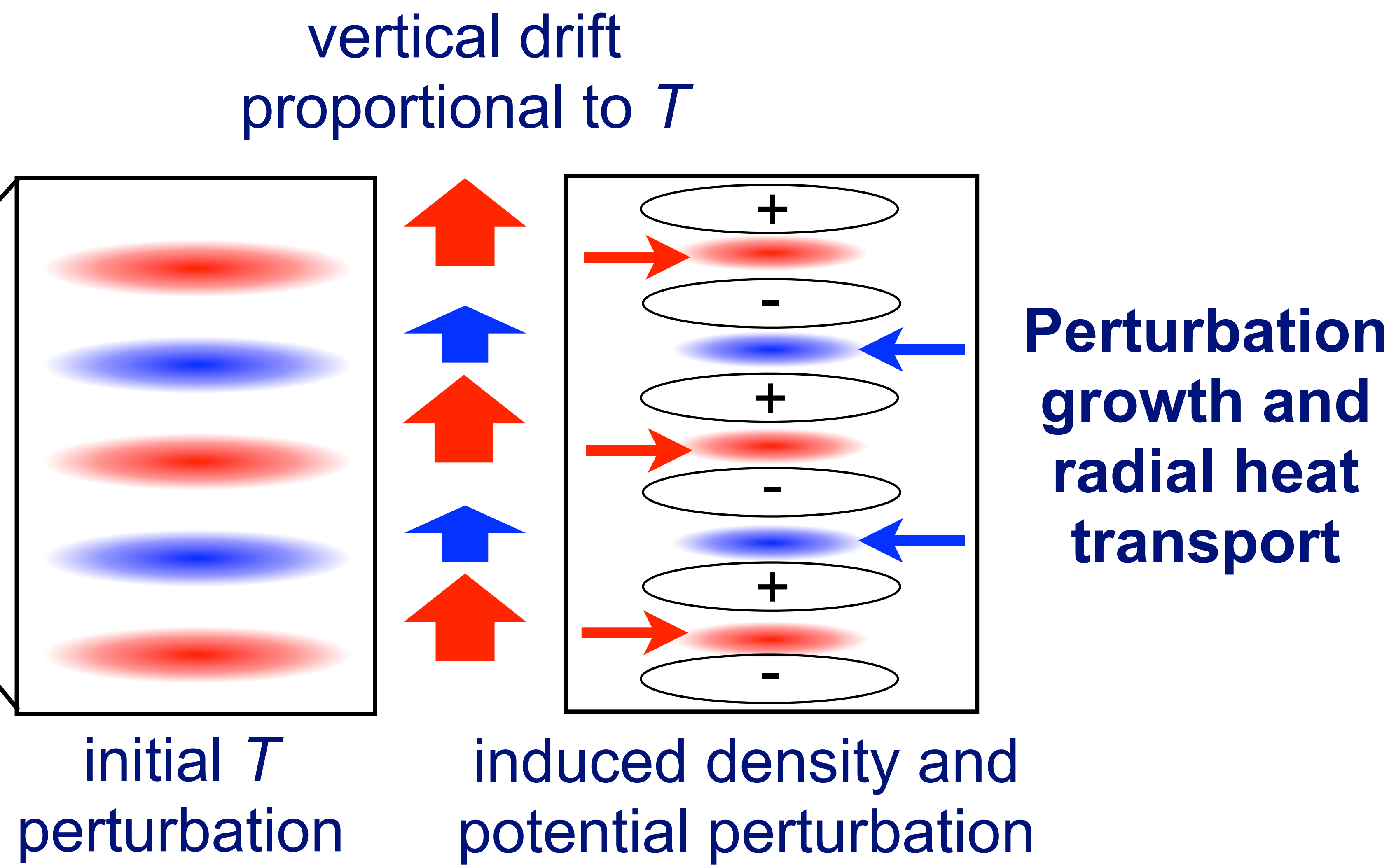


- Potential perturbation implies ExB drift:

$$\mathbf{v}_{E\times B} = \frac{\mathbf{b}\times\nabla\phi}{B}$$

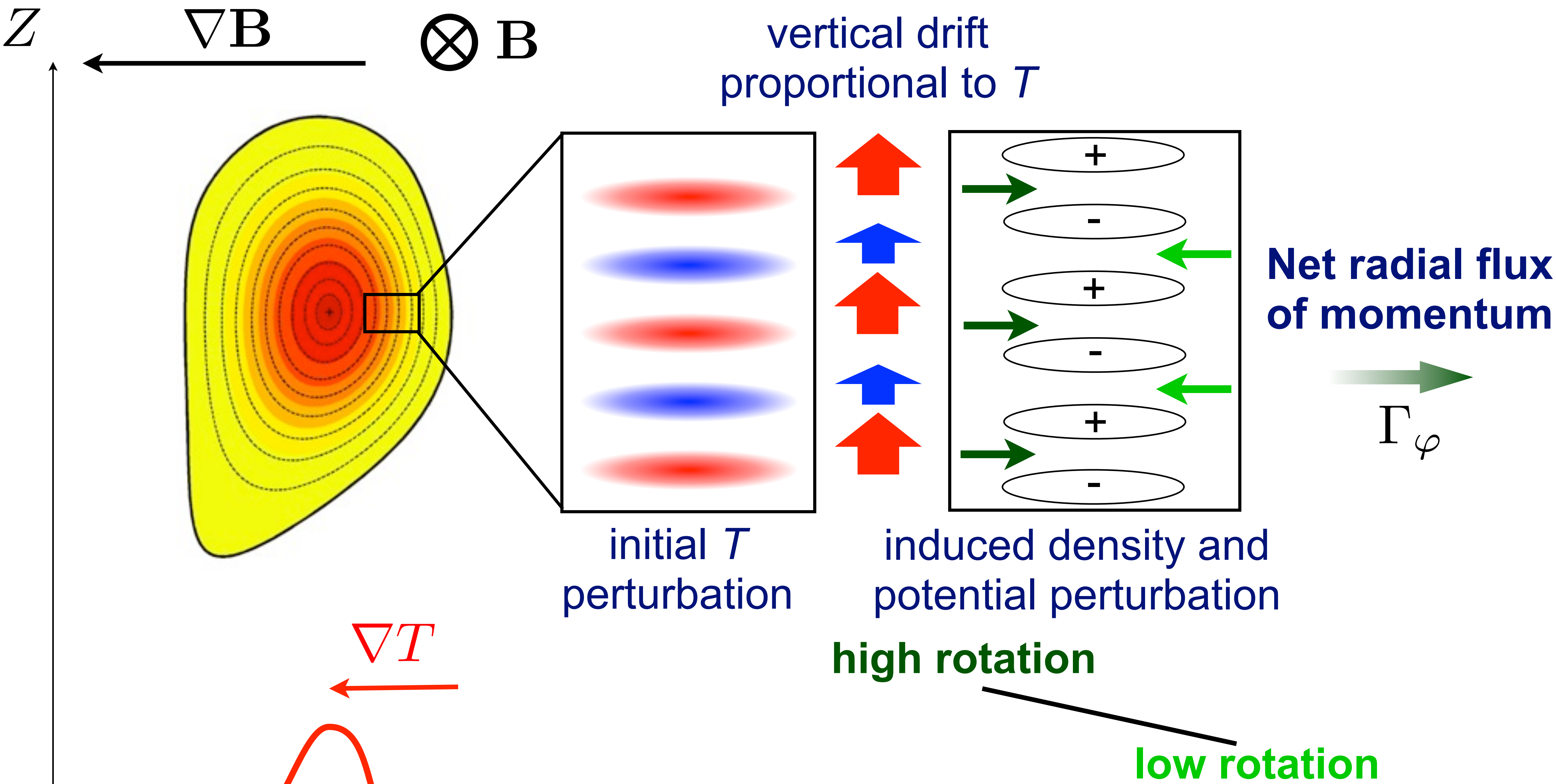
Temperature gradient → instability&transport

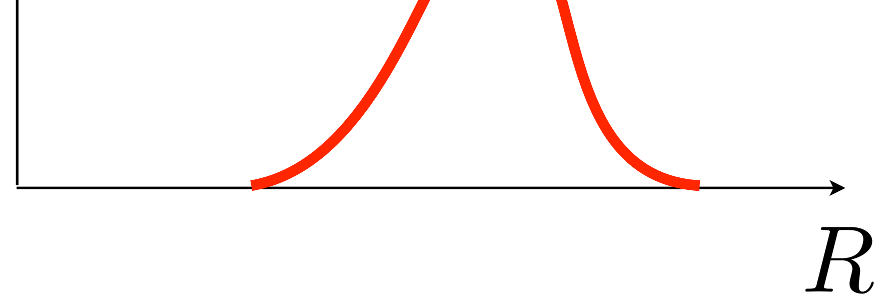




- Because of the temperature gradient hot plasma is convected into the hot spots and cold plasma into the cold spots

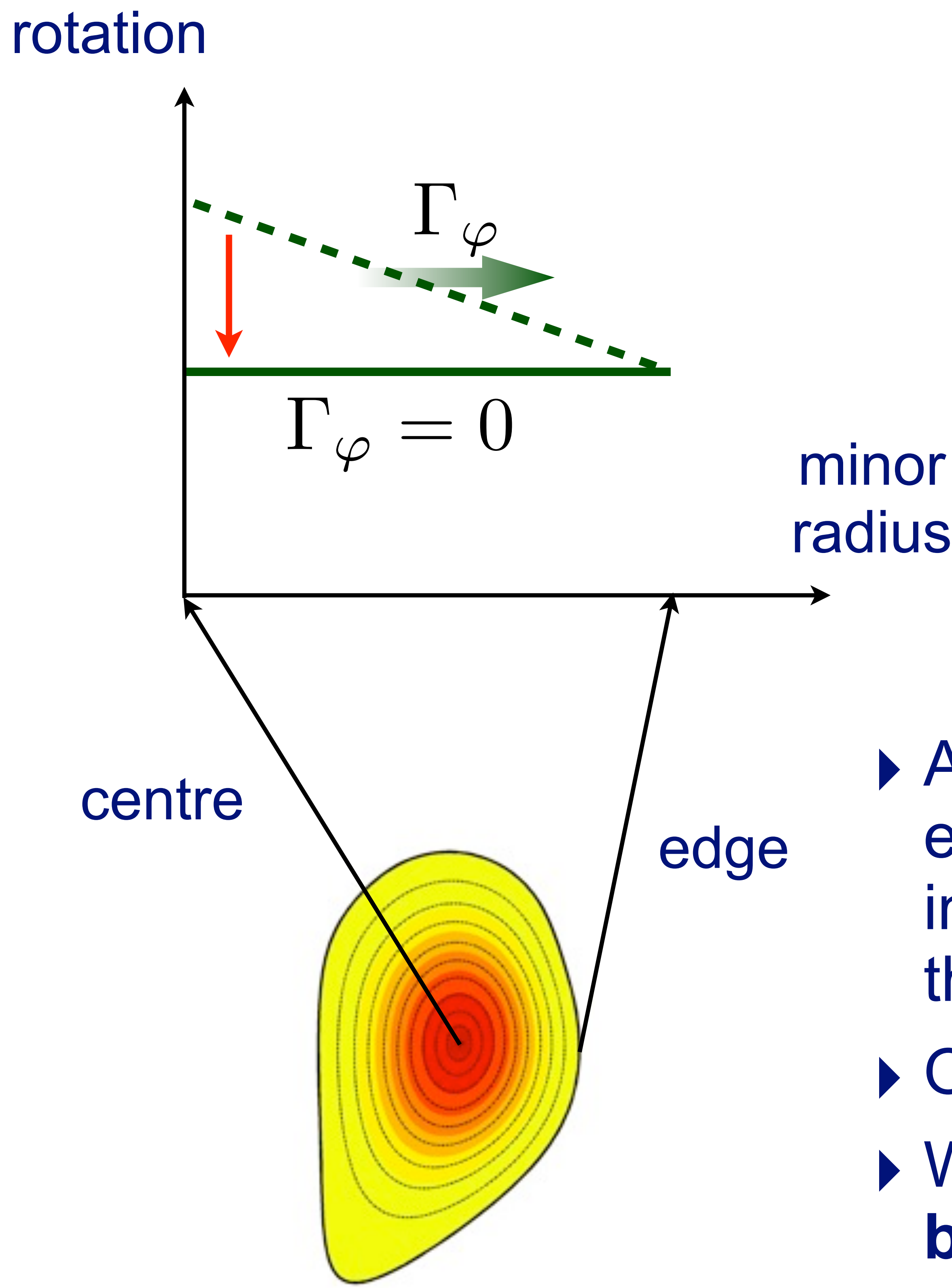
What about rotation?





- Finite rotation gradient and ITG turbulence
- ExB convection relaxes the rotation gradient

Diag. momentum flux → flat rotation profile



radial flux of momentum

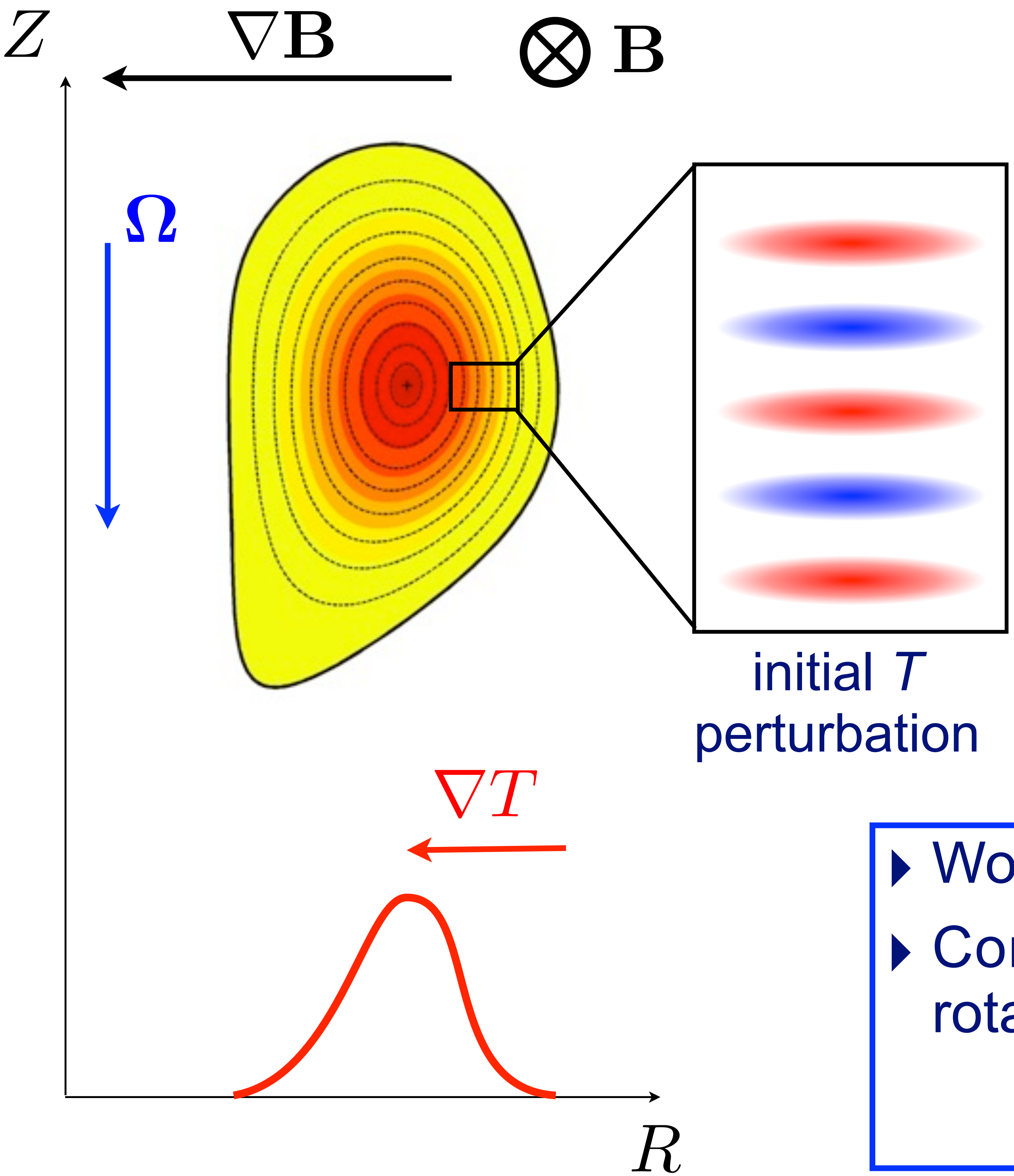
toroidal rotation gradient

$$\boxed{\Gamma_\varphi = -nmR_0\chi_\varphi \frac{\partial u_\varphi}{\partial r}}$$

momentum diffusivity

- As soon as a toroidal rotation gradient exists, the radial flux of momentum induced by the ITG turbulence will relax this gradient
- Only stationary **flat profiles** are possible
- Whole rotation profile determined by the **boundary conditions**

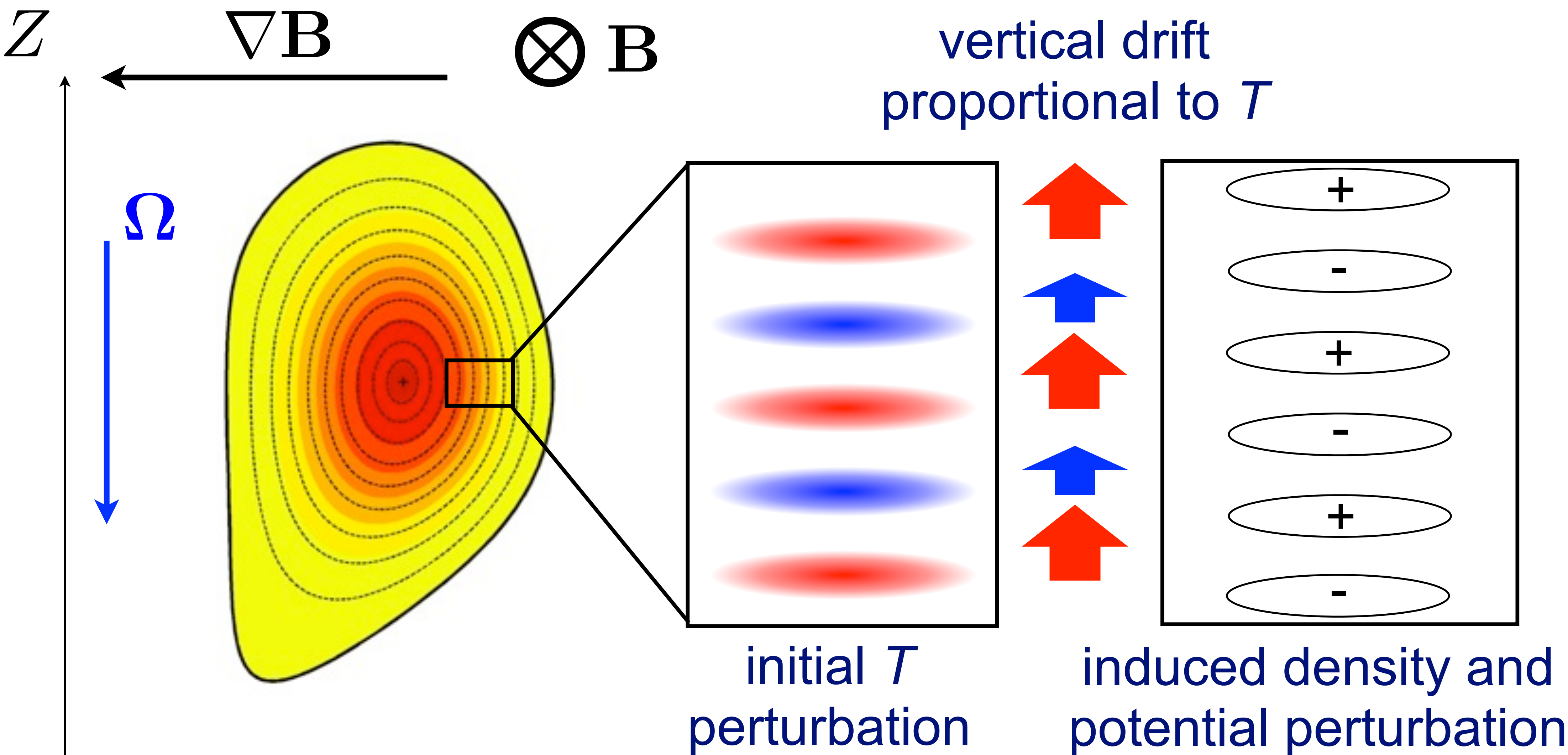
Same initial picture + background rotation

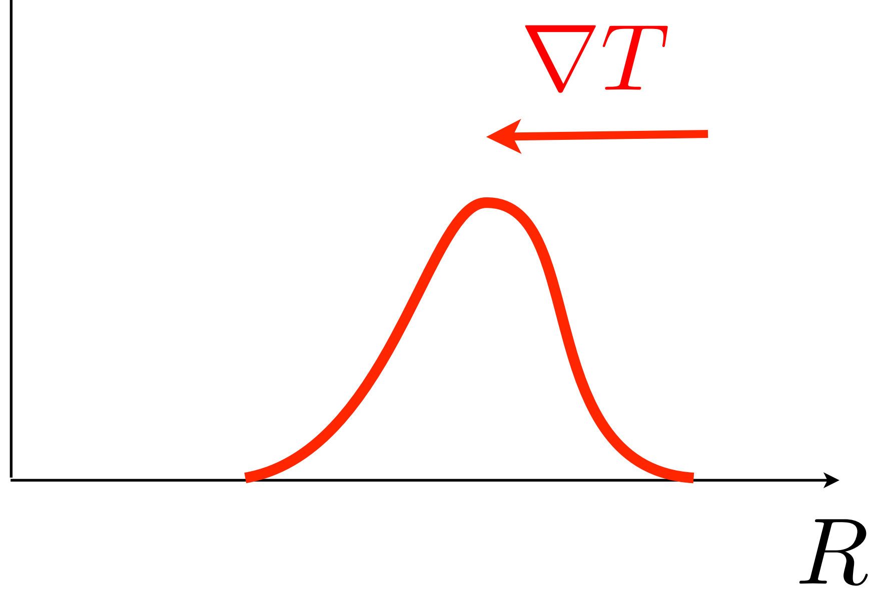


- Work in the co-moving frame
- Coriolis force due to toroidal rotation:

$$\mathbf{F}_{\mathrm{co}} = 2m\mathbf{v} \times \mathbf{\Omega}$$

Coriolis drift vertical and proportional to $v_{//}$





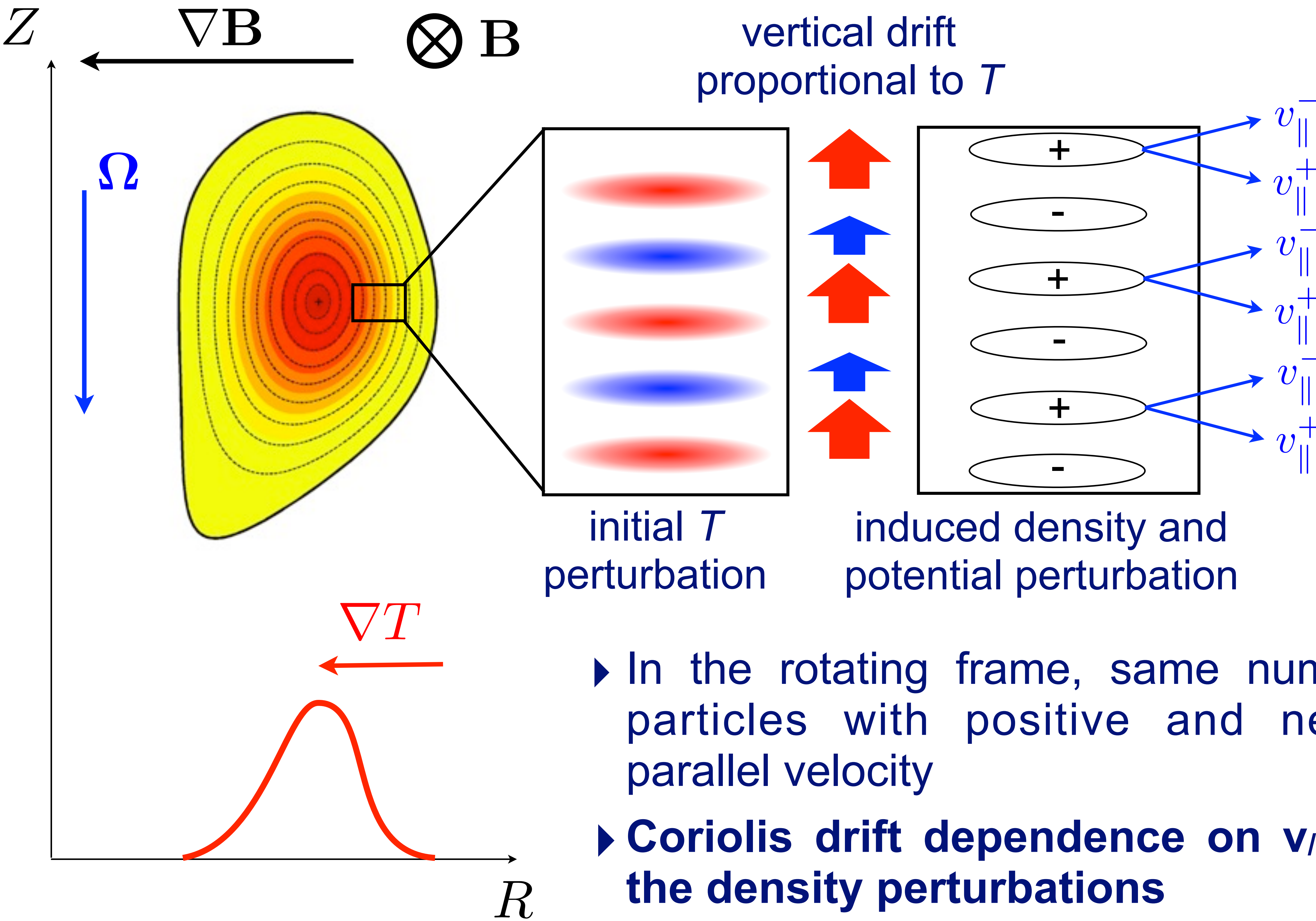
Coriolis drift:

$$\mathbf{v}_{co} = 2\frac{mv_\parallel}{ZeB}\mathbf{\Omega}_\perp$$

$\uparrow \quad v_\parallel < 0$

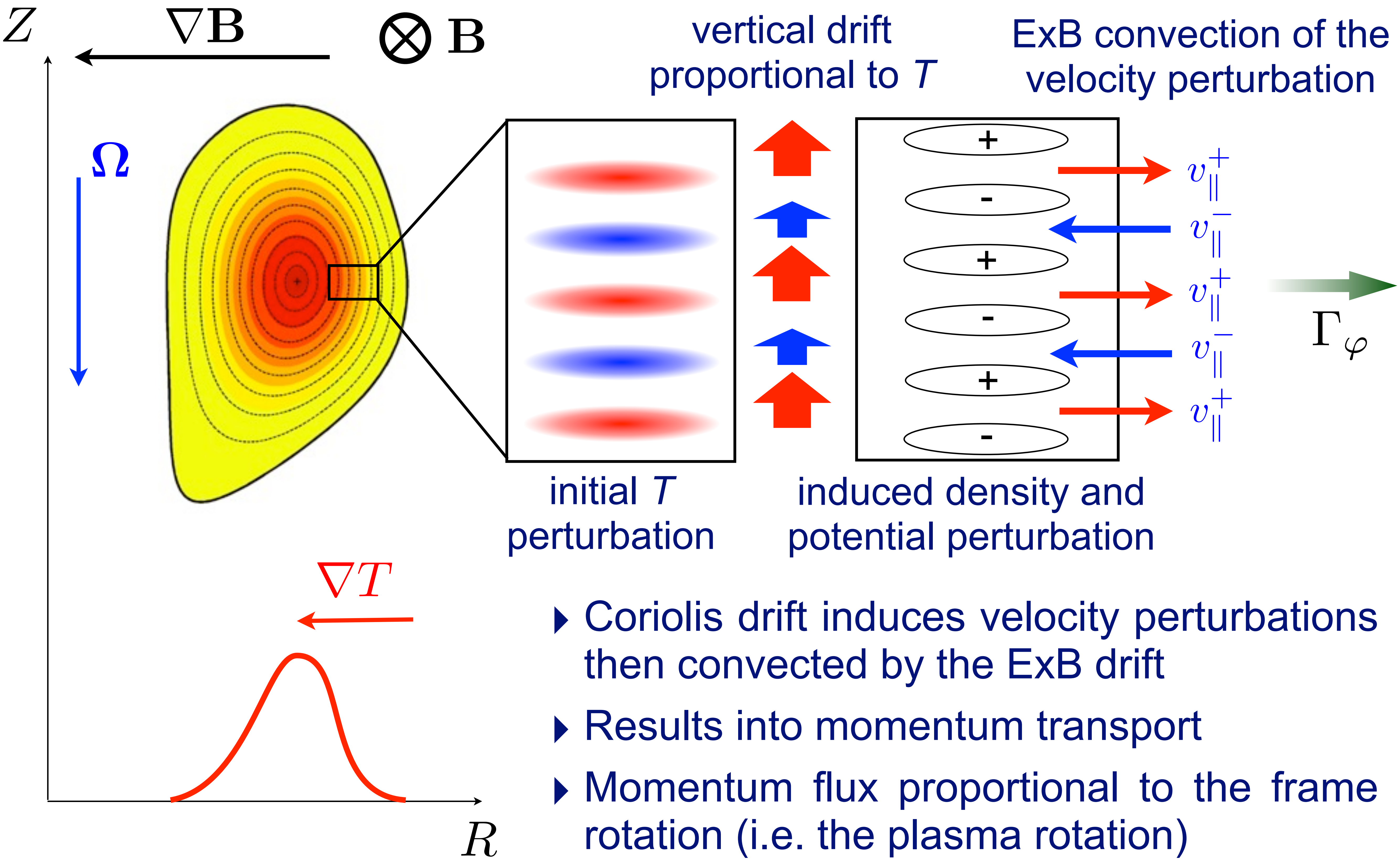
$\downarrow \quad v_\parallel > 0$

Coriolis drift dependence on v// is the key!



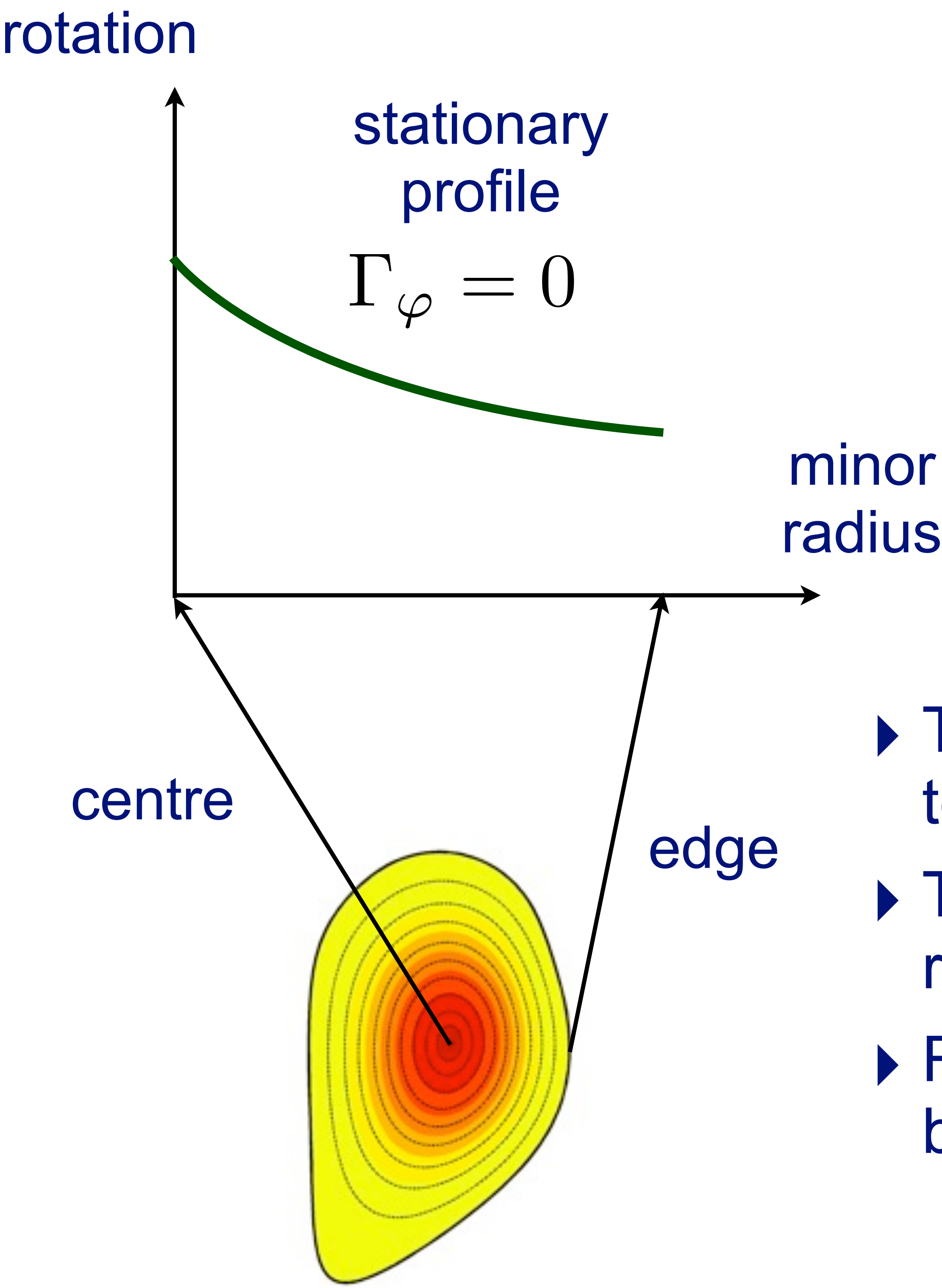
- In the rotating frame, same number of particles with positive and negative parallel velocity
- **Coriolis drift dependence on v// splits the density perturbations**

Coriolis drift leads to momentum transport



- Coriolis drift induces velocity perturbations then convected by the ExB drift
- Results into momentum transport
- Momentum flux proportional to the frame rotation (i.e. the plasma rotation)

Coriolis pinch → peaked rotation profile



radial flux of momentum

toroidal rotation

$$\boxed{\Gamma_\varphi = nmR_0[-\chi_\varphi \frac{\partial u_\varphi}{\partial r} + V_{co} u_\varphi]}$$

pinch coefficient

- The Coriolis pinch (generally inward) tends to enhance the core rotation
- The diagonal part tends to relax the rotation gradient
- Rotation profile given by the balance between the two terms

General picture

- Parallel symmetry breaking with respect to the midplane required to get turbulent momentum flux
- Symmetry breaking by:
  - Toroidal rotation gradient $\rightarrow$ diagonal flux
  - Toroidal rotation $\rightarrow$ Coriolis pinch
  - Others $\rightarrow$ residual stress
- Generic expression for the turbulent momentum flux:

$$\Gamma_\varphi = nmR_0[\underbrace{-\chi_\varphi \frac{\partial u_\varphi}{\partial r}}_{\text{diagonal}} + \underbrace{V_{co} u_\varphi}_{\text{pinch}} + \underbrace{C_\varphi}_{\text{residual stress}}]$$

- All terms tend to matter for intrinsic rotation!!

[Last overview: Peeters NF'11]

Typical tokamak values

[Last overview: Peeters NF'11]

- Normalise with thermal velocity v_{th}:

$$\Gamma_\varphi = nmv_{\rm th}\chi_\varphi[-\underbrace{\frac{R_0}{v_{\rm th}}\frac{\partial u_\varphi}{\partial r}}_{1\text{-}3} + \frac{R_0 V_{co}}{\chi_\varphi}\underbrace{\frac{u_\varphi}{v_{\rm th}}}_{0.1\text{-}0.3} + \frac{R_0 C_\varphi}{v_{\rm th}\chi_\varphi}]$$

- Prandtl number: $\boxed{\chi_\varphi/\chi_i \sim 0.6-1}$ exp/th
- Pinch number: $\boxed{RV_{\rm co}/\chi_\phi \sim 1-5}$ exp/th
  - generally inward
  - scales with the trapped particle fraction
- Stress number: $\boxed{C^*/\chi_\phi \sim 0-1}$ mainly th, few exp quantitative studies
  - inward or outward, many components
  - can change sign at the ITG/TEM transition *[Camenen NF'11]*

Part II - NTV

- Now, forget about turbulence and assume small non-axisymmetric perturbations of the magnetic field, e.g. ripple

$$\frac{\partial}{\partial t} < mnR\mathbf{e}_\varphi \cdot \mathbf{u} > \sim \; - < R\mathbf{e}_\varphi \cdot \nabla \cdot \underline{\underline{\pi}}_{i\|} > - \frac{1}{V'} \frac{\partial}{\partial r} \cancel{\left[V' < \Pi_{r\varphi}^{\mathrm{turb}} > \right]}$$

NTV

turbulence

$$\sim < (p_\| - p_\perp) \frac{1}{B} \frac{\partial B}{\partial \varphi} > \quad \cancel{\sim < nmR\tilde{u}_r \tilde{u}_\varphi + mRu_\varphi \tilde{u}_r \tilde{n} >}$$

- **In the following:**
  - A word on trapped particles orbits in rippled tokamaks
  - Impact on the rotation profile
  - Big brush picture

Rippled tokamaks

- Broken toroidal symmetry due to finite number *N* of toroidal field coils

- Magnetic field given by:

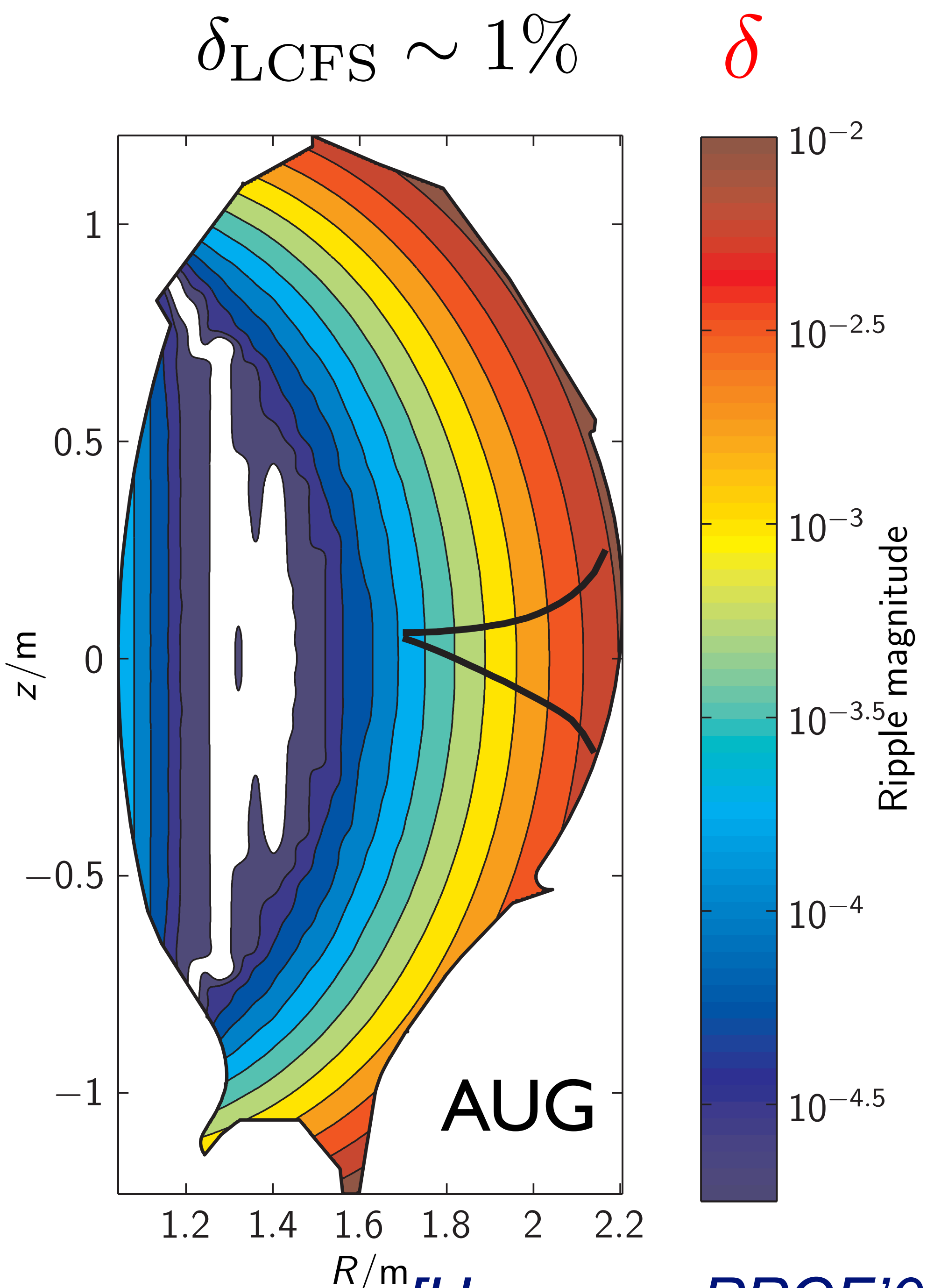
$$B = B_0[1 - \epsilon \cos\theta][1 - \underbrace{\delta(r,\theta)}_{\text{ripple amplitude}} cos N\varphi]$$

- In tokamaks, typically:

$$\boxed{\delta_{\mathrm{LCFS}} = 0.1\% - 5\%}$$

JET: 0.1 - 1%
TCV/AUG: 1%
Tore Supra: 5-6%

$\delta_{\mathrm{LCFS}} \sim 1\%$



[Hynonen PPCF'08]

Locally trapped particles

- Broken toroidal symmetry due to finite number *N* of toroidal field coils

$$B = B_0[1 - \epsilon\cos\theta][1 - \delta(r,\theta)\cos N\varphi]$$

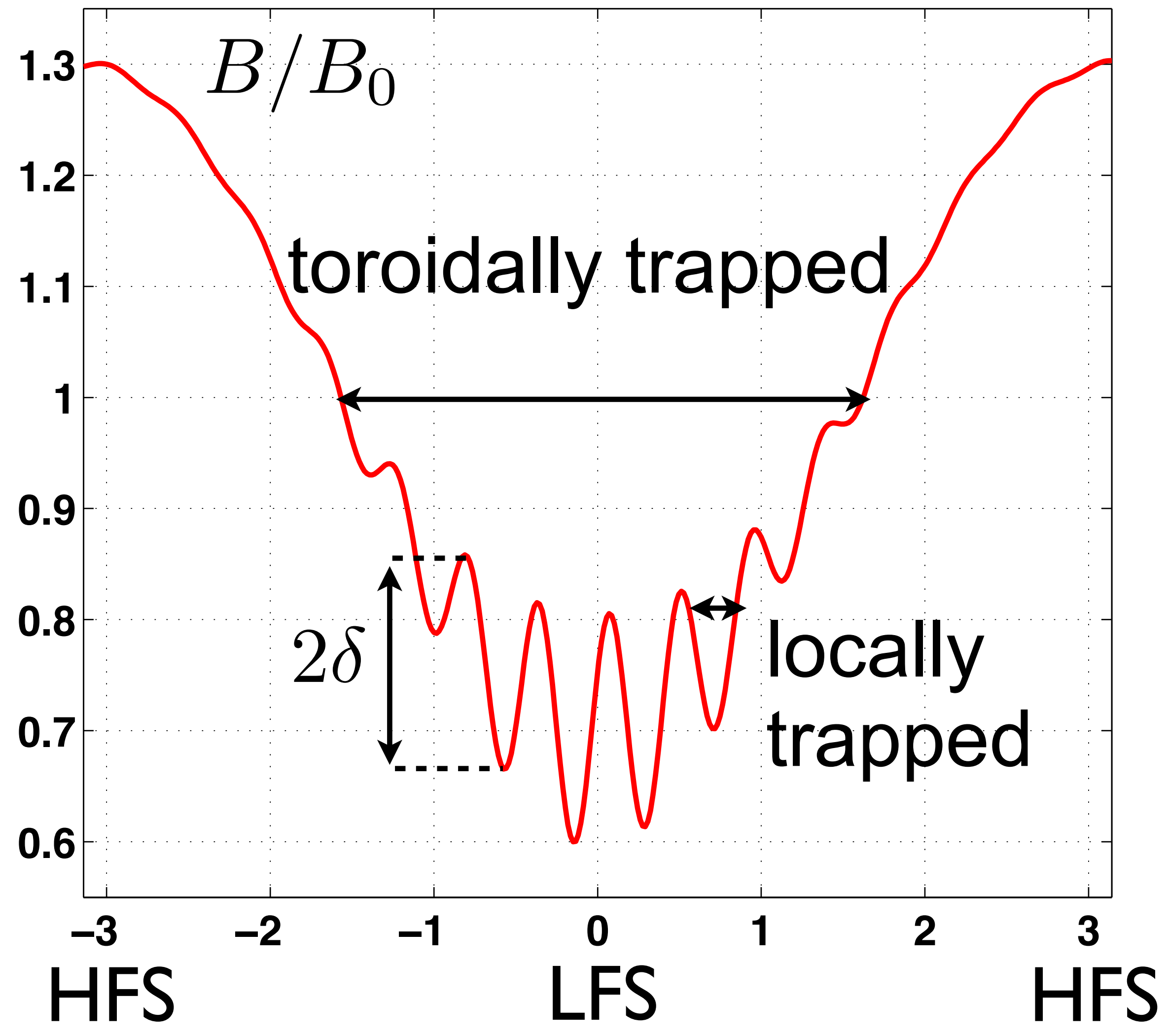
- Along a field line, $\varphi = \varphi_0 + q\theta$, local extrema in B may exist

- Particles can be trapped in these local magnetic wells

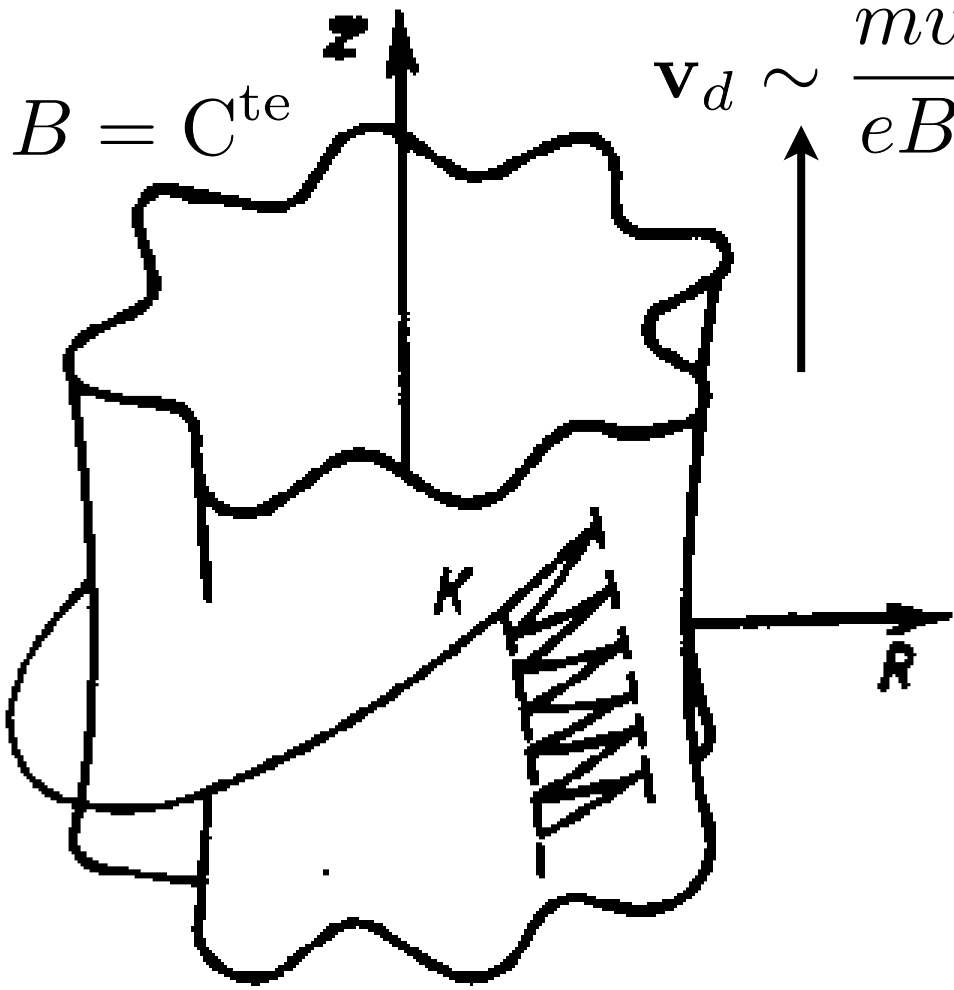
- Local wells exist if:

$$\boxed{Y = \frac{\epsilon}{Nq\delta}|\sin\theta| < 1}$$

RATHER SMALL
REGION IN TOKAMAKS



Locally trapped particles



$B = \mathrm{C}^{\mathrm{te}}$

$\mathbf{v}_d \sim \frac{m v_{\mathrm{th}}^2}{eBR} \mathbf{e}_z$

[Yushmanov RPP'90]

- Oscillations in the local wells: *ripple*

$$v_\parallel \sim v_{\mathrm{th}} \sqrt{\delta} \qquad L \sim R/N$$

$$\longrightarrow \quad \tau_l \sim R/(N v_{\mathrm{th}} \sqrt{\delta})$$

- Vertical drift until they escape the local well (or the plasma)

+ toroidicity

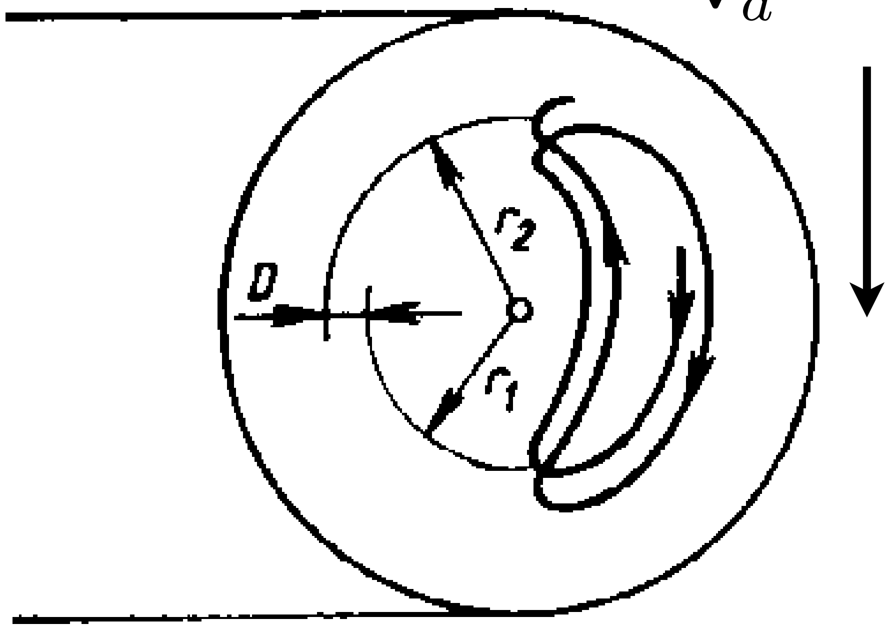
- Radial diffusion with collisional detrapping:

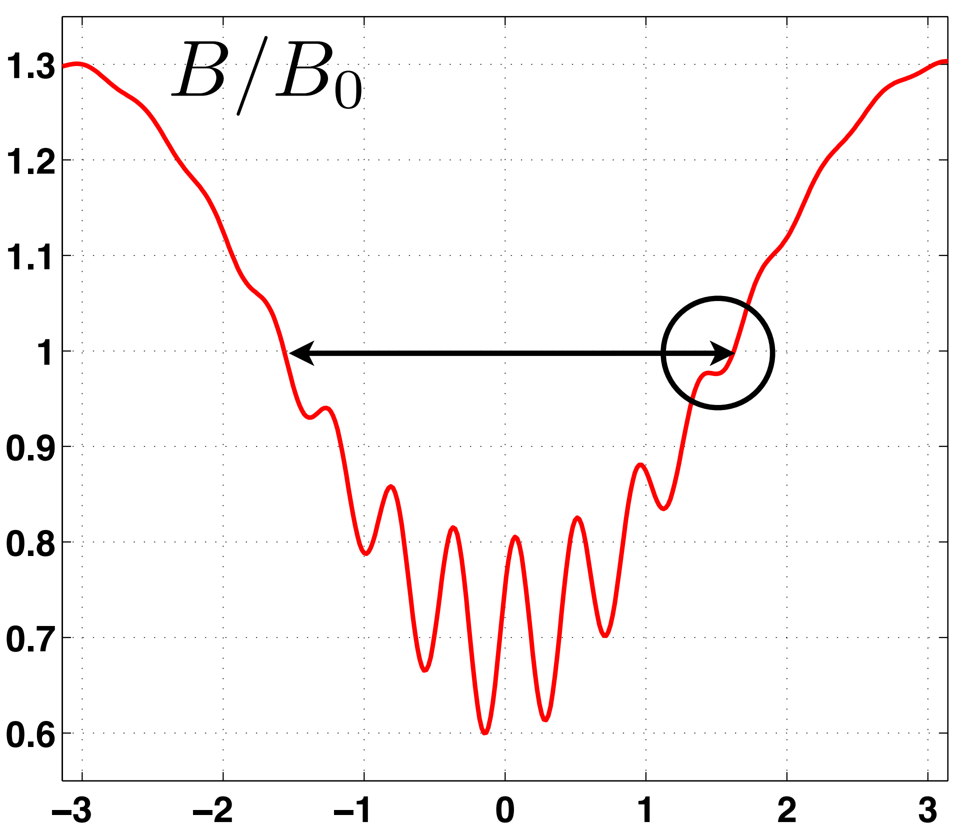
$$\underbrace{\sim \sqrt{\delta}}_{n_l} \cdot \underbrace{\left[\frac{v_d}{\nu_{\mathrm{eff}}} \right]}_{\Delta r}^2 \cdot \underbrace{\nu_{\mathrm{eff}}}_{\nu/\delta} = \delta^{3/2} \frac{v_d^2}{\nu}$$

- Very bad for high energy particles...
- Can be effectively decreased by ExB drift

Ripple also modifies banana orbits

$$\mathbf{v}_d \sim \frac{m v_{\rm th}^2}{eBR}\mathbf{e}_z$$





- Banana bounce time: *toroidicity*

$$v_\parallel \sim v_{\rm th}\sqrt{\epsilon} \qquad L \sim qR$$

$$\longrightarrow \quad \tau_t \sim qR/(v_{\rm th}\sqrt{\epsilon})$$

- Parallel velocity modified by ripple:

$$v_\parallel = \sqrt{2\mu/m}\sqrt{B_{\rm bounce} - \bar{B} - \tilde{B}}$$

+ ripple

- Especially effective near bounce point
- Radial excursion at banana tip:

$$\sim \rho(q/\epsilon)^{3/2}\delta\sqrt{N}$$

[Yushmanov RPP'90]

Impact on rotation?

- Ripple modifies particle trajectories

  $\longrightarrow$ enhanced radial particle flux

- This enhanced particle flux is species dependent (non-ambipolar)

  $\longrightarrow$ radial current

- Exerts a JxB torque on the plasma

  $\longrightarrow$ toroidal acceleration

- Stops when Er makes the particle flux ambipolar

- How large a torque for a given non-ambipolar diffusion?

$$\underline{mnR\frac{\partial u_t}{\partial t}} = R\mathbf{e}_\varphi \cdot \mathbf{j} \times \mathbf{B} = Rj_r B_p$$

$$j_r \sim e\Gamma^{\mathrm{na}} \sim eD^{\mathrm{na}}\frac{n}{a}$$

$$B_p \sim B\epsilon/q$$

torque density [N.m/m^3] $\sim \frac{neB}{q}D^{\mathrm{na}}$ $\longrightarrow$ 0.2m^2/s gives ~1-2 N.m/m^3

[1NBI source ~ 1-2N.m]

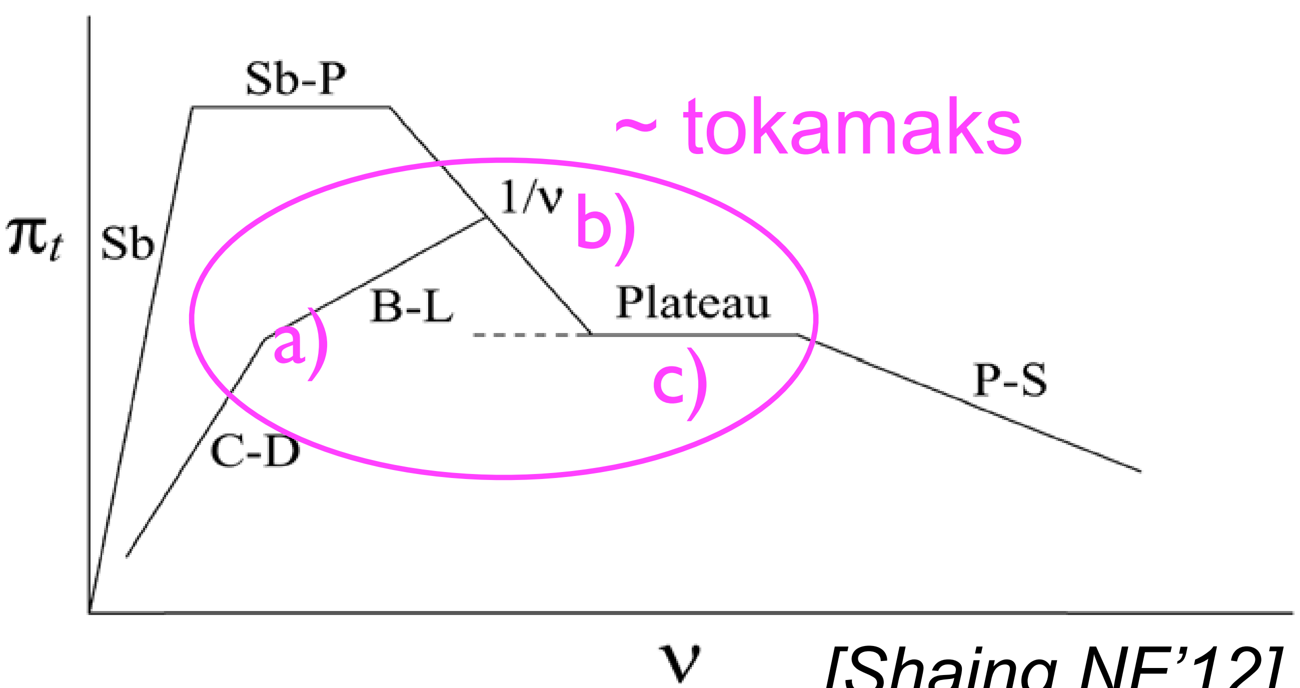
NTV: general picture & estimates

- Generic form of the NTV term:

$$< R\mathbf{e}_\varphi \cdot \nabla \cdot \underline{\underline{\pi}}_{i\|} > \sim nm\underbrace{\nu_{\rm na}}_{\text{damping rate}} (< Ru_t > - \underbrace{< Ru_t^{\rm neo} >}_{\text{offset rotation}})$$

$$\sim k_{\rm T} \frac{1}{eB_p} \frac{\partial T}{\partial r}$$

- Kinetic approach required (at least at low collisionality)
- Can be computed analytically in various limits
  - Very useful to highlight the physics (many different regimes!!)



[Shaing NF'12]

	$\nu_{\rm na}$	$k_{\rm T}$
a)	$1.1\sqrt{\epsilon}\nu$	3.54
b)	$\propto \frac{\delta^2\epsilon^{3/2}}{Nq^3}\frac{v_{\rm th}^2}{R^2}\frac{1}{\nu}$	3.54
c)	$\propto \delta^2 N \frac{v_{\rm th}}{R}$	1.67

[Garbet NF'09]

- Ultimately numerical simulations required *[e.g. Sun PRL'10, Satake PRL'11]*

Part III - NTV and turbulent transport

- Stationary rotation profile given by:

$$0 = - < R\mathbf{e}_\varphi \cdot \nabla \cdot \underline{\underline{\pi}}_{i\|} > - \frac{1}{V'} \frac{\partial}{\partial r} \left[V' < \Pi_{r\varphi}^{\mathrm{turb}} > \right]$$

- Which roughly gives:

$$r\nu_{\mathrm{na}}[u_\varphi - u_\varphi^{\mathrm{neo}}] = -\frac{\partial}{\partial r} \left[r[-\chi_\varphi \frac{\partial u_\varphi}{\partial r} + V_{co} u_\varphi + C_\varphi] \right]$$

NTV turbulence

- Now, the question is:
"Is there a dominant term or do we need to keep all??"

- Let's assume the toroidal flow is u_φ^{neo} and look how large a turbulent flux it would drive

NTV versus turbulence

$$r\nu_{\rm na}[u_\varphi - u_\varphi^{\rm neo}] = -\frac{\partial}{\partial r}\left[r[-\chi_\varphi \frac{\partial u_\varphi}{\partial r} + V_{co} u_\varphi + C_\varphi]\right]$$

NTV turbulence

- Simplified calculation:

$$u_\varphi^{\rm neo} = \frac{k_T}{eB}\frac{q}{\epsilon}\frac{\partial T}{\partial r}$$

$$R/L_T = \frac{R}{T}\frac{\partial T}{\partial r} = C^{\rm te}$$

$$\longrightarrow$$

$$\frac{\partial u_\varphi^{\rm neo}}{\partial r} \sim \frac{u_\varphi^{\rm neo}}{R} R/L_T$$

$$\frac{\partial^2 u_\varphi^{\rm neo}}{\partial r^2} \sim \frac{u_\varphi^{\rm neo}}{R^2}(R/L_T)^2$$

- Neglecting residual stress, the divergence of the flux is then:

$$\text{turb} \sim [1 + \epsilon R/L_T][R/L_T + \frac{RV_{\rm co}}{\chi_\varphi}]\frac{\chi_\varphi}{R} u_\varphi^{\rm neo}$$

- Which remains to be compared to the NTV drag rate (ripple-plateau):

$$\text{NTV} \sim \epsilon\delta^2 N v_{\rm th}[u_\varphi - u_\varphi^{\rm neo}]$$

NTV versus turbulence

- Some numbers:

$$\text{turb} \sim \underline{[1+\epsilon R/L_T][R/L_T+\frac{RV_{\text{co}}}{\chi_\varphi}]\frac{\chi_\varphi}{R}} u_\varphi^{\text{neo}}$$

[1.5 - 4].[7 - 14].[1 - 4] → 10 - 200 m.s^{-1}

$$\text{NTV} \sim \underline{\epsilon\delta^2 N} v_{\text{th}}[u_\varphi - u_\varphi^{\text{neo}}]$$

[0.1 - 0.3].[0.5% - 2%]^2.[16 - 20].[3e5 - 1e6] → 10 - 2000 m.s^{-1}

- In many cases, NTV drag and turbulent transport can be expected to have a comparable effect on the stationary rotation profile!

Summary

- **Many** physical mechanisms can affect toroidal rotation:
  - In an axisymmetric tokamak, turbulent transport provides a few candidates (especially for residual stress)
  - Break the toroidal symmetry and NTV will give you even more of them
- Rough estimates indicate that turbulent transport and NTV will often have a comparable effect on the stationary rotation profile
- This is not the full story:
  - the boundary condition (friction on neutrals, CX losses, orbit losses...) is at least as important.
  - difference between impurity (measured) and bulk rotation is likely non negligible
- Toroidal rotation physics is definitely complex...
  Makes our life a bit difficult but also provides more knobs to control the resulting profile (and to explain the wealth of puzzling experimental observations)

Starting point: moment equations

[No particle or momentum sources, for simplicity]

- density:

$$\boxed{\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{u} = 0} \qquad \text{with} \qquad n = \int f \,\mathrm{d}\mathbf{v} \quad \text{and} \quad \mathbf{u} = \frac{1}{n}\int \mathbf{v} f \,\mathrm{d}\mathbf{v}$$

- momentum:

$$\boxed{mn\frac{\partial \mathbf{u}}{\partial t} + mn\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p - \nabla\cdot\underline{\underline{\pi}} + Zen\left[\mathbf{E} + \mathbf{u}\times\mathbf{B}\right] + \mathbf{R}^{\mathrm{col}}}$$

$$\text{with} \qquad p = nT = \frac{1}{3}\int m(\mathbf{v}-\mathbf{u})^2 f \,\mathrm{d}\mathbf{v}$$

$$\underline{\underline{\pi}} = \int m(\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u}) f \,\mathrm{d}\mathbf{v} - p\underline{\underline{I}}$$

- heat: ...
- Neoclassical and turbulent contributions included:

$$f = \bar{f}_0 + \underbrace{\bar{f}_1 + \ldots}_{\text{NTV}} + \underbrace{\tilde{f}_1 + \ldots}_{\text{turbulence}}$$